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Distributed Data-Driven Coordination of IBRs for Grid-Level Optimal Voltage Control

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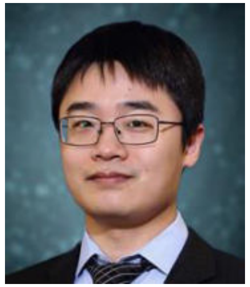

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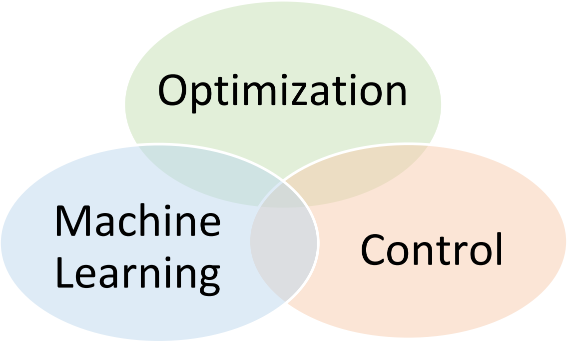
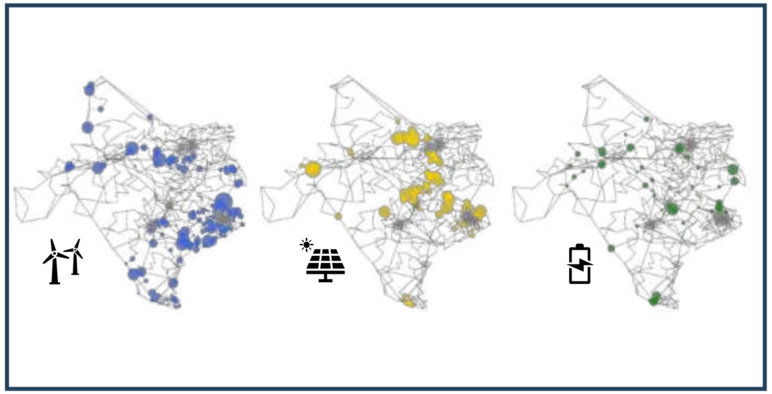
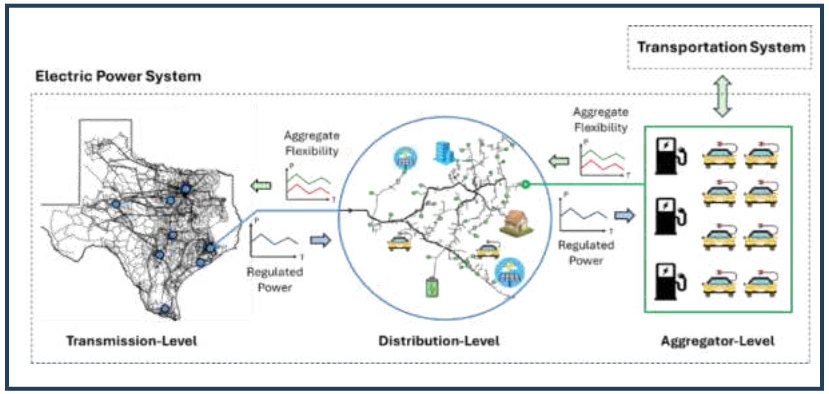


Smart Power & Energy Systems

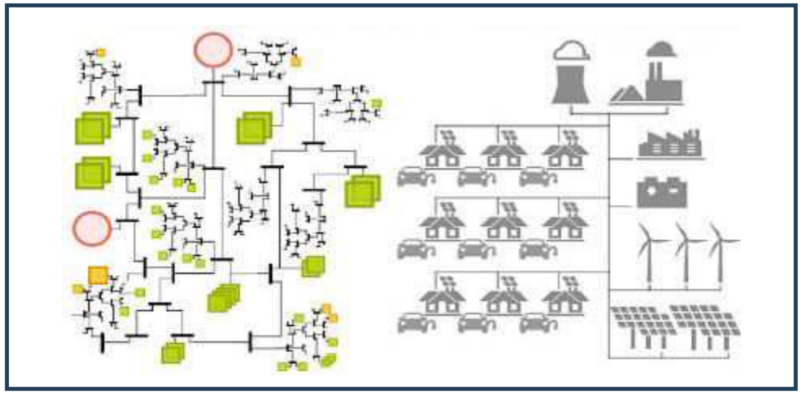
1. Large-Scale Power Systems Optimization Under Uncertainty



2. Autonomous Coordination of Massive Flexible DERs

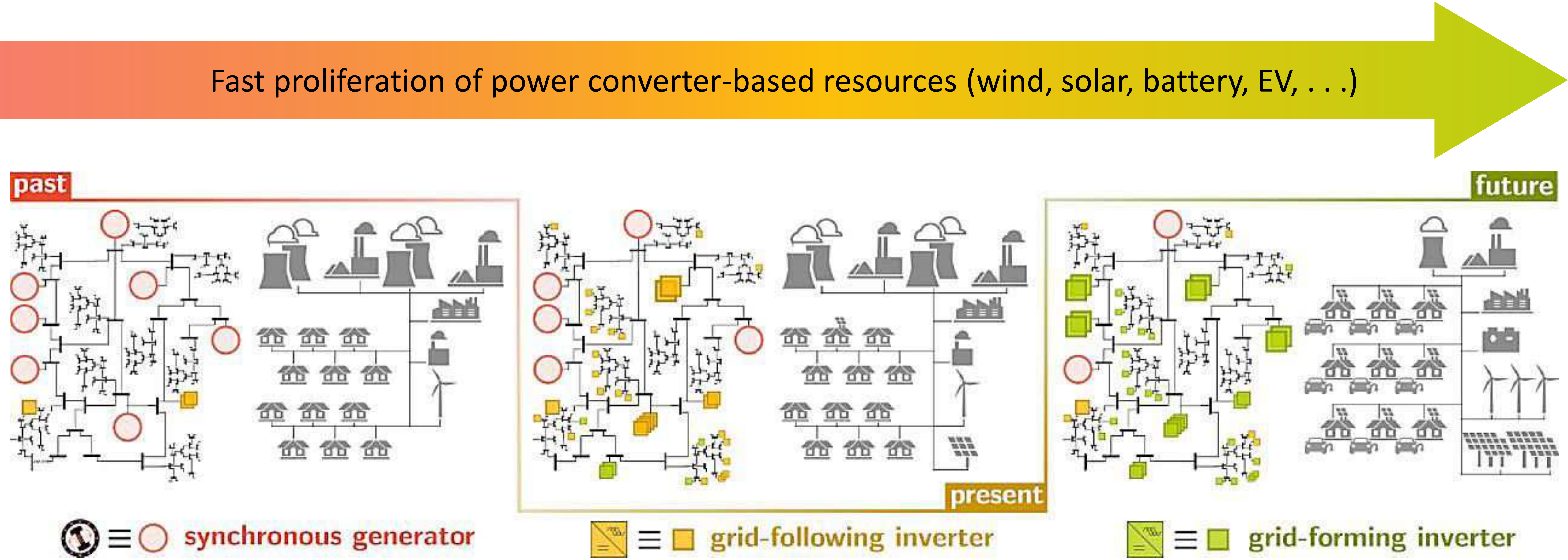


3. Data-Driven Physics-Informed Modeling and Control of IBRs



Scalable Learning-Based Control and Optimization Theories, Algorithms, and Tools

Transformation of Power Systems



How to coordinate large-scale inverter-based resources (IBRs) in the grid?

Image source: https://unificonsortium.org/about/

Key Challenges

Scalability

- Millions of heterogenous IBRs
- Large-scale power networks
- Limited communication & computation

Unknown Model

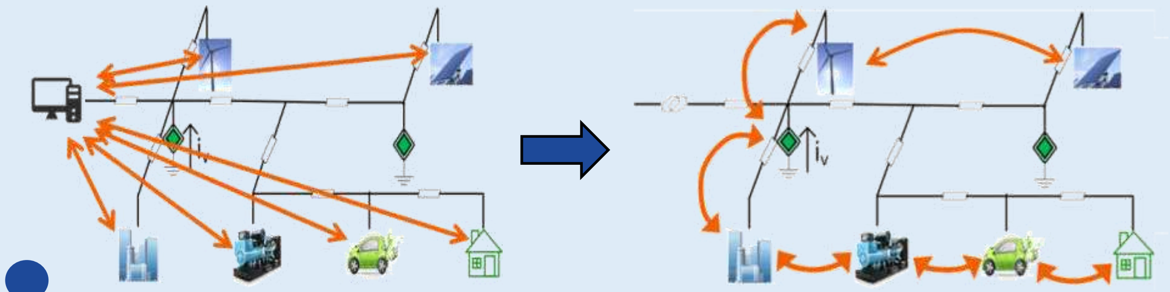
- Complex physical grid dynamics
- Uncertain renewables and loads
- Unknown IBR dynamics models . . .

Our Solutions

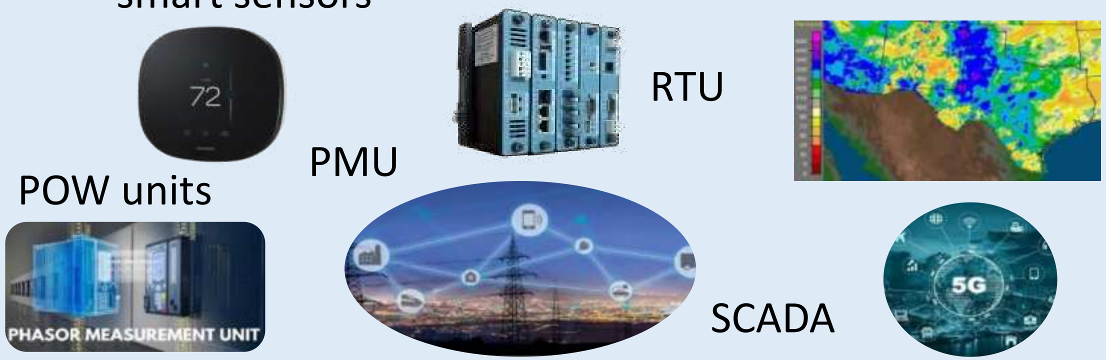
"Leverage specific problem structures for control design"

Centralized Control

Distributed Control

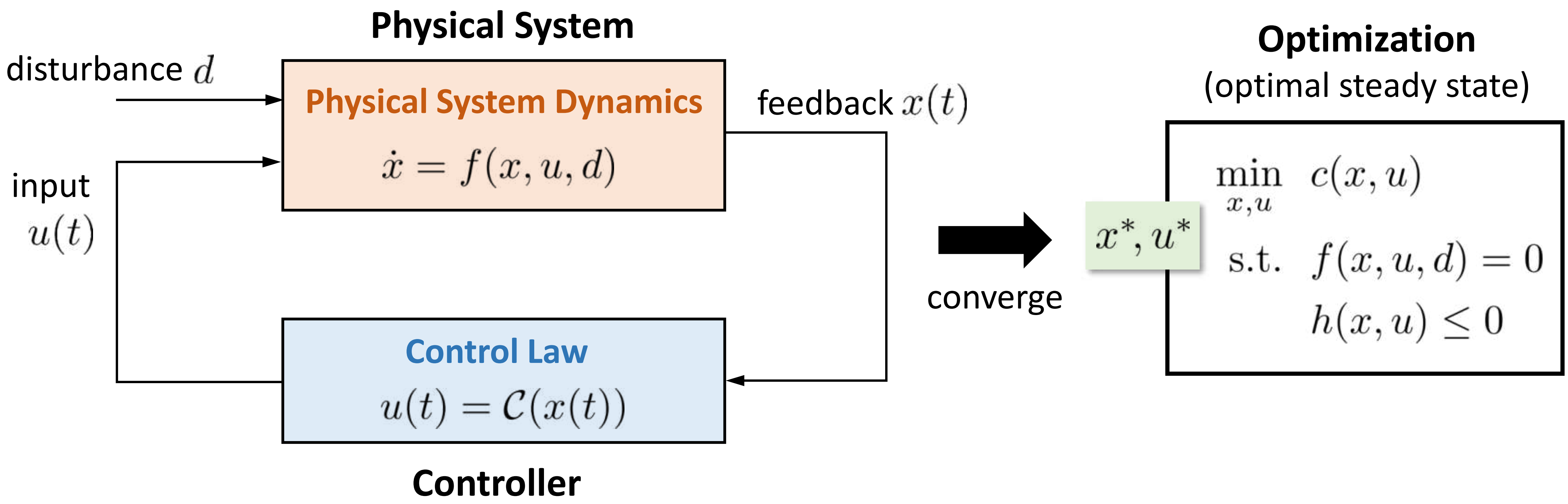


Data-Driven/AI-Based Control

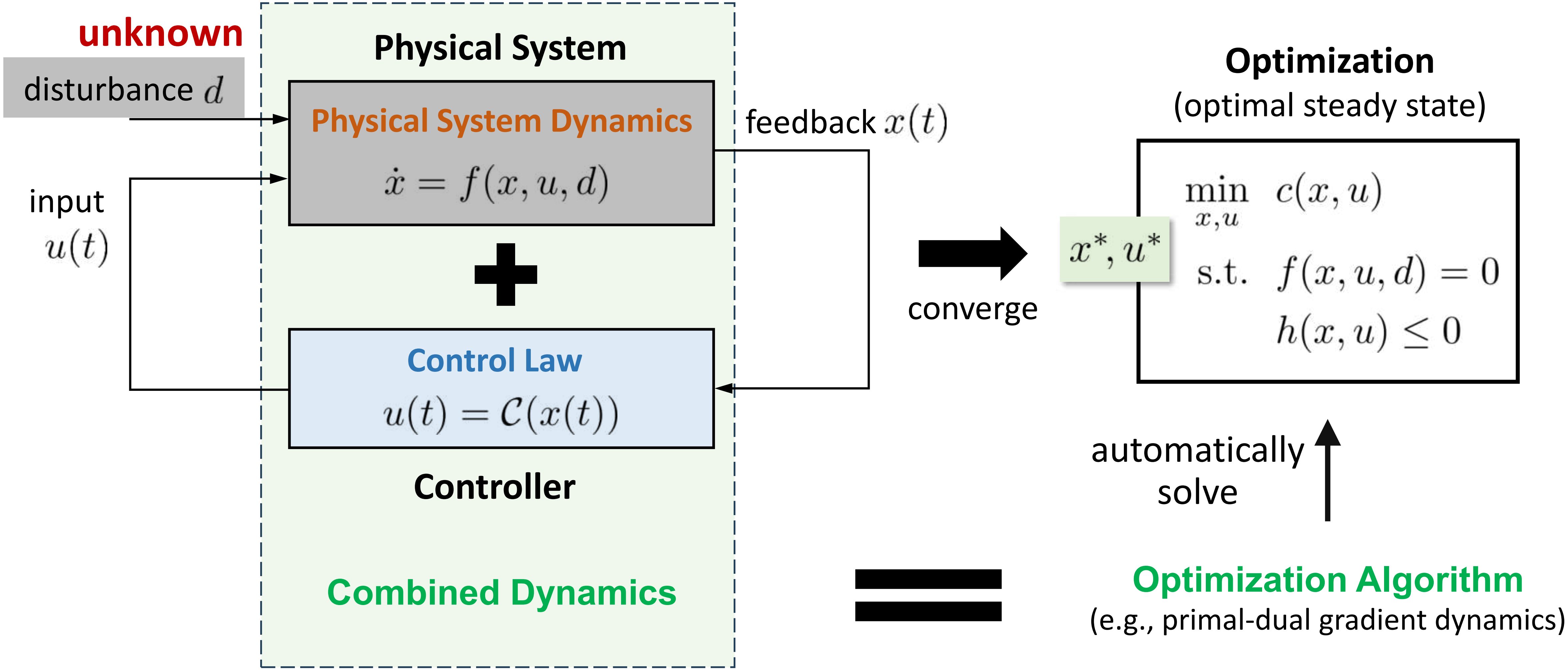


RTU: remote terminal unit, POW: point-on-wave, PMU: phasor measurement unit, SCADA: supervisory control and data acquisition

Key Idea: Feedback Optimization

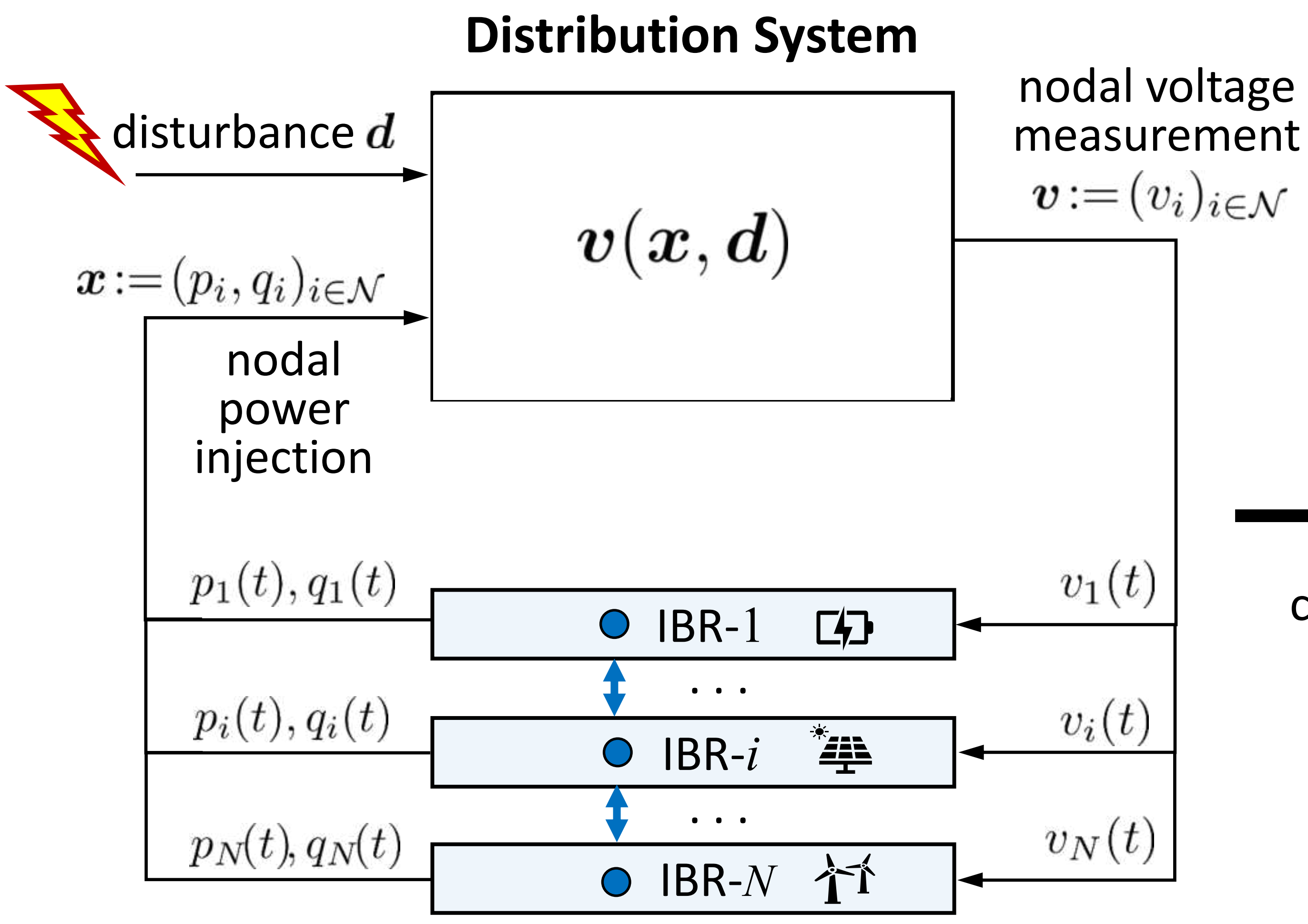


Key Idea: Feedback Optimization



[**Chen**, Li, ACC 2020] [**Chen**, Zhao, Li, TCNS 2021] [**Chen**, Poveda, Li, CDC 2021] [**Chen**, Poveda, Li, arXiv, 2023]

Real-Time Optimal Voltage Control (OVC)



Control **IBR Units**

converge $\boldsymbol{x}_i^*$

- ✓ **Control Optimality**
- ✓ **Voltage Safety**

decision $\boldsymbol{x}_i := (p_i, q_i)$

control cost

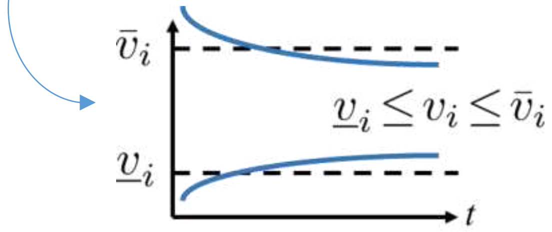
OVC

$$\text{Obj.} \quad \min_{\boldsymbol{x}} \sum_{i \in \mathcal{N}} c_i(\boldsymbol{x}_i)$$

IBR power feasible set

$$\text{s.t.} \quad \boldsymbol{x}_i \in \mathcal{X}_i, \quad i \in \mathcal{N}$$

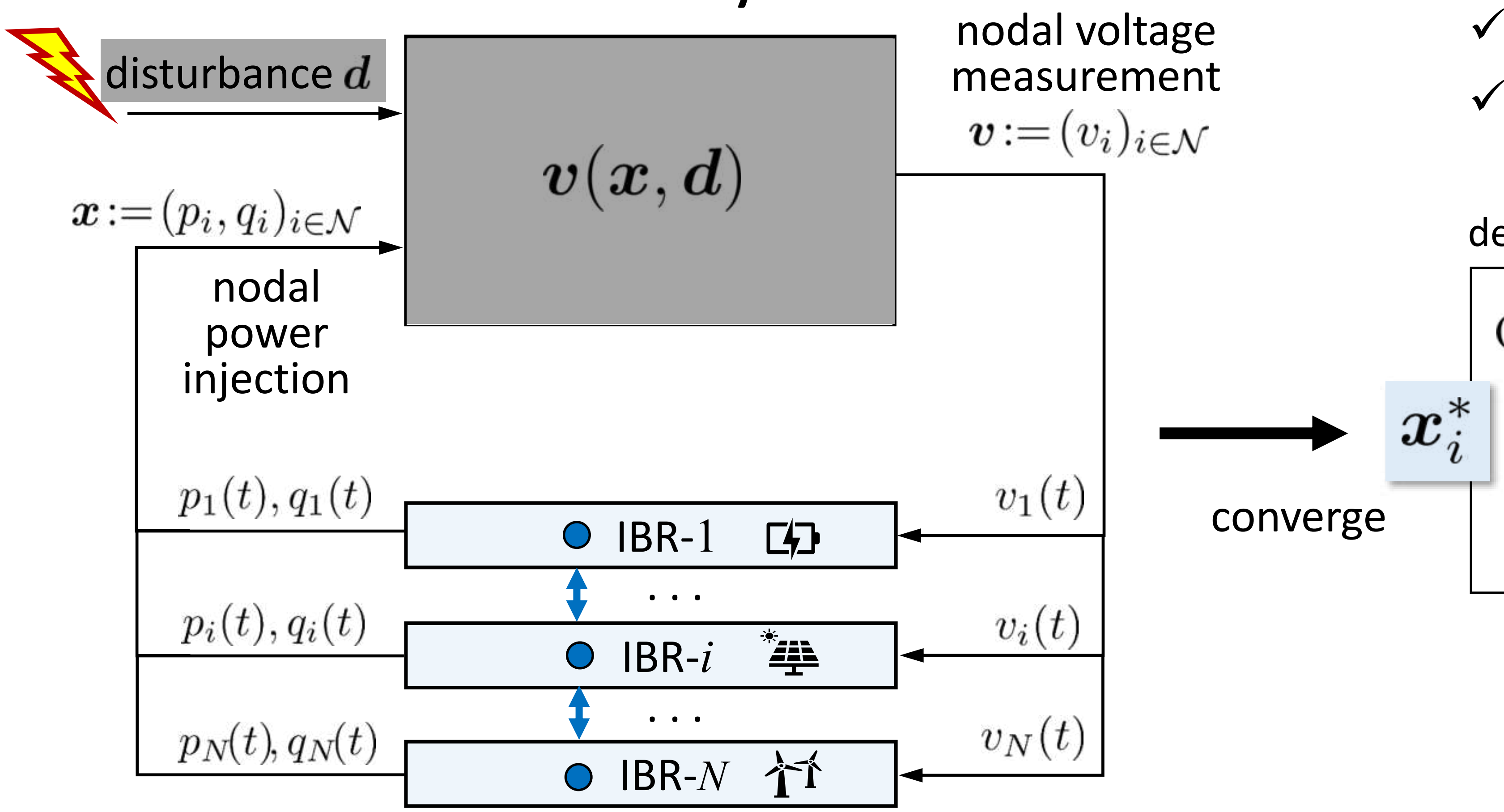
$$\underline{v}_i \leq v_i(\boldsymbol{x}, \boldsymbol{d}) \leq \bar{v}_i, \quad i \in \mathcal{N}$$



voltage safety interval

Real-Time Optimal Voltage Control (OVC)

Unknown Model **Distribution System**



- ✓ **Control Optimality**
- ✓ **Voltage Safety**

decision $\boldsymbol{x}_i := (p_i, q_i)$

OVC

$$
\begin{aligned}
\text{Obj.} \quad & \min_{\boldsymbol{x}} \sum_{i \in \mathcal{N}} c_i(\boldsymbol{x}_i) \\
\text{s.t.} \quad & \boldsymbol{x}_i \in \mathcal{X}_i, && i \in \mathcal{N} \\
& \underline{v}_i \le v_i(\boldsymbol{x}, \boldsymbol{d}) \le \bar{v}_i, && i \in \mathcal{N}
\end{aligned}
$$

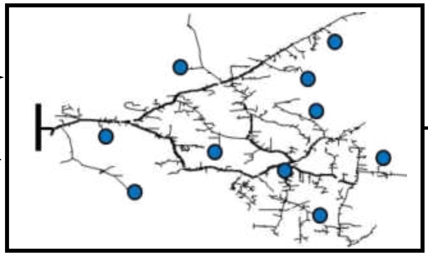
? ***Control*** **IBR Units**

Scalability: massive IBRs

Optimal Voltage Control Design

Distribution System

disturbance $\boldsymbol{d}$



voltage measurement

power injection

$\boldsymbol{x}_i := (p_i, q_i)$

P-PDGD

$v_i^{\mathrm{m}}(t)$

IBR i

converge

$\boldsymbol{x}_i^*$

OVC

$$\text{Obj.} \quad \min_{\boldsymbol{x}} \sum_{i \in \mathcal{N}} c_i(\boldsymbol{x}_i)$$

$$\text{s.t.} \quad \boldsymbol{x}_i \in \mathcal{X}_i, \qquad i \in \mathcal{N}$$

$$\underline{v}_i \le v_i(\boldsymbol{x}, \boldsymbol{d}) \le \bar{v}_i, \qquad i \in \mathcal{N}$$

Projected Primal-Dual Gradient Dynamics (P-PDGD)

$$\dot{\boldsymbol{x}}_i = k_x \left[\mathrm{Proj}_{\mathcal{X}_i} \left(\boldsymbol{x}_i - \alpha_x (\nabla c_i(\boldsymbol{x}_i) + \boldsymbol{\ell}_i) \right) - \boldsymbol{x}_i \right]$$

$$\dot{\lambda}_i^+ = k_\lambda \left[\mathrm{Proj}_{\mathbb{R}_+} (\lambda_i^+ + \alpha_\lambda (v_i(\boldsymbol{x}, \boldsymbol{d}) - \bar{v}_i)) - \lambda_i^+ \right]$$

$$\dot{\lambda}_i^- = k_\lambda \left[\mathrm{Proj}_{\mathbb{R}_+} (\lambda_i^- + \alpha_\lambda (\underline{v}_i - v_i(\boldsymbol{x}, \boldsymbol{d}))) - \lambda_i^- \right]$$

$$\boldsymbol{\ell}_i = \sum_{j \in \mathcal{N}} (\lambda_j^+ - \lambda_j^-) \frac{\partial v_j(\boldsymbol{x}, \boldsymbol{d})}{\partial \boldsymbol{x}_i}$$

$i \in \mathcal{N}$

solved by

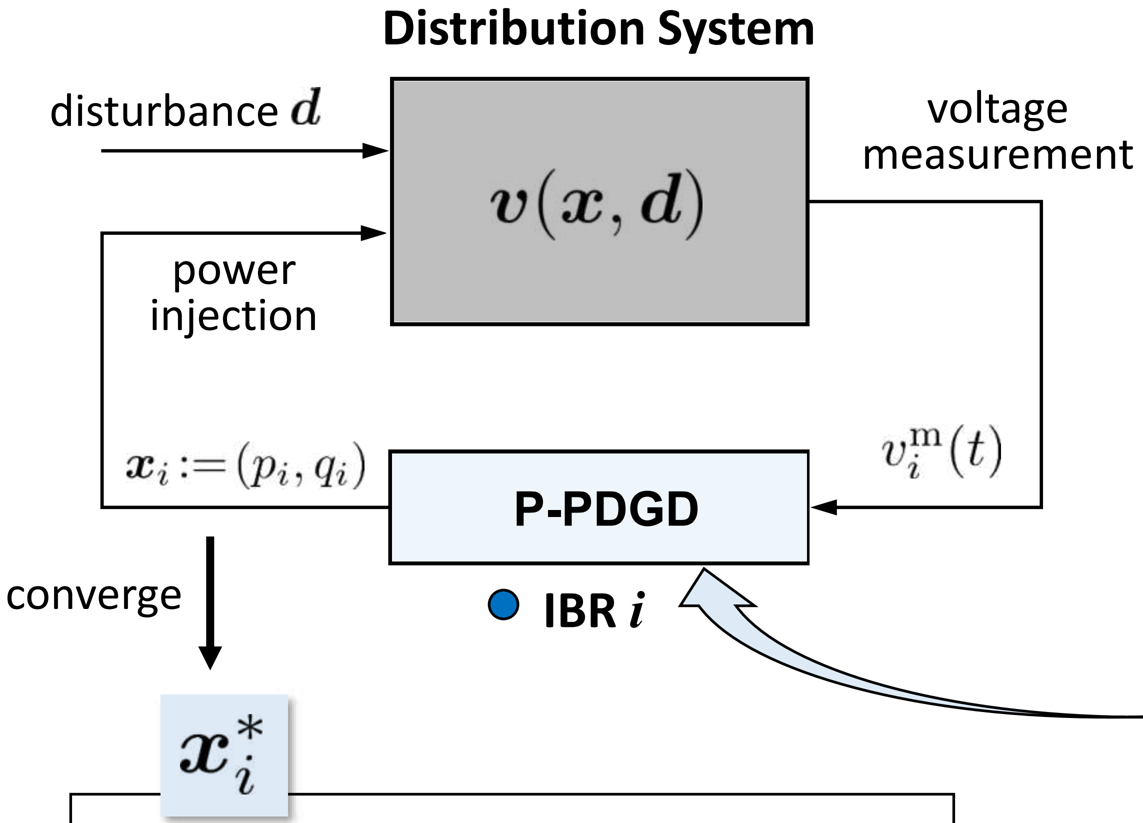
Saddle Point Problem

dual variable

$\boldsymbol{\lambda} := (\lambda_i^+, \lambda_i^-)_{i \in \mathcal{N}}$

$$\max_{\boldsymbol{\lambda} \ge 0} \min_{\boldsymbol{x} \in \mathcal{X}} L(\boldsymbol{x}, \boldsymbol{\lambda}) = \sum_{i \in \mathcal{N}} c_i(\boldsymbol{x}_i) + \sum_{i \in \mathcal{N}} \left[\lambda_i^+ (v_i(\boldsymbol{x}, \boldsymbol{d}) - \bar{v}_i) + \lambda_i^- (\underline{v}_i - v_i(\boldsymbol{x}, \boldsymbol{d})) \right]$$

Optimal Voltage Control Design



Projected Primal-Dual Gradient Dynamics (P-PDGD)

$$\dot{\boldsymbol{x}}_i = k_x \left[\mathrm{Proj}_{\mathcal{X}_i} \left(\boldsymbol{x}_i - \alpha_x (\nabla c_i(\boldsymbol{x}_i) + \boldsymbol{\ell}_i) \right) - \boldsymbol{x}_i \right]$$

$$\dot{\lambda}_i^+ = k_\lambda \left[\mathrm{Proj}_{\mathbb{R}_+} (\lambda_i^+ + \alpha_\lambda (v_i(\boldsymbol{x}, \boldsymbol{d}) - \bar{v}_i)) - \lambda_i^+ \right]$$

$$\dot{\lambda}_i^- = k_\lambda \left[\mathrm{Proj}_{\mathbb{R}_+} (\lambda_i^- + \alpha_\lambda (\underline{v}_i - v_i(\boldsymbol{x}, \boldsymbol{d}))) - \lambda_i^- \right]$$

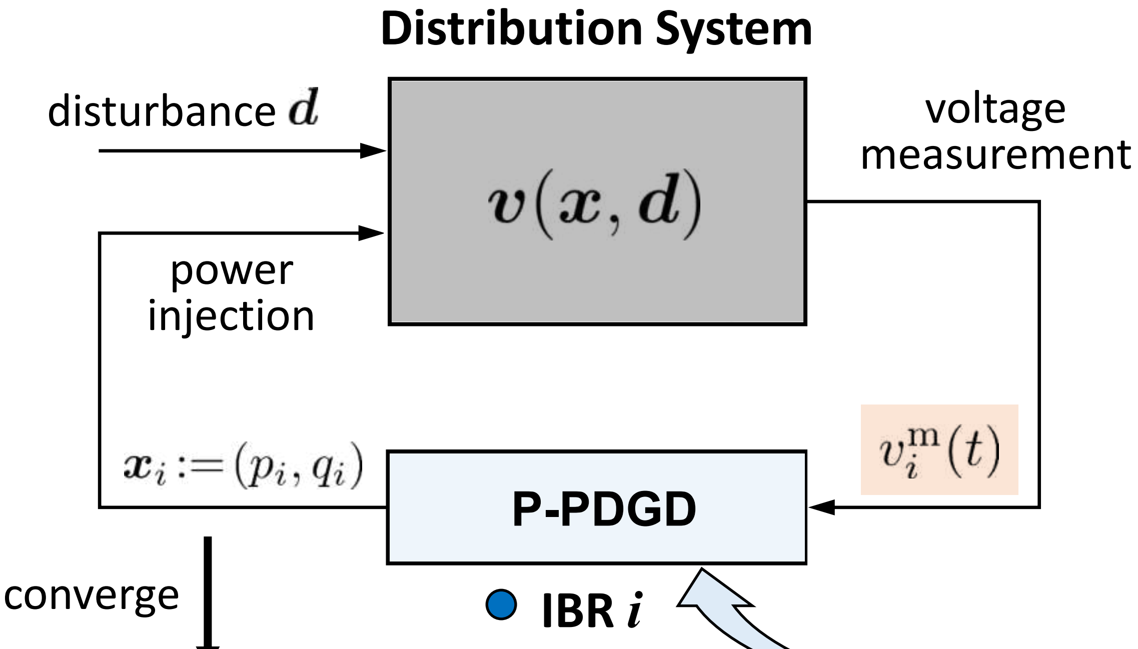
$$\boldsymbol{\ell}_i = \sum_{j \in \mathcal{N}} (\lambda_j^+ - \lambda_j^-) \frac{\partial v_j(\boldsymbol{x}, \boldsymbol{d})}{\partial \boldsymbol{x}_i} \qquad i \in \mathcal{N}$$

not implementable
due to **unknown system model**

OVC

$$\begin{aligned} \text{Obj.} \quad & \min_{\boldsymbol{x}} \sum_{i \in \mathcal{N}} c_i(\boldsymbol{x}_i) & \\ \text{s.t.} \quad & \boldsymbol{x}_i \in \mathcal{X}_i, & i \in \mathcal{N} \\ & \underline{v}_i \le v_i(\boldsymbol{x}, \boldsymbol{d}) \le \bar{v}_i, & i \in \mathcal{N} \end{aligned}$$

Optimal Voltage Control Design



$\boldsymbol{x}_i^*$

OVC

$$
\begin{aligned}
\text{Obj.} \quad & \min_{\boldsymbol{x}} \sum_{i \in \mathcal{N}} c_i(\boldsymbol{x}_i) & \\
\text{s.t.} \quad & \boldsymbol{x}_i \in \mathcal{X}_i, & i \in \mathcal{N} \\
& \underline{v}_i \leq v_i(\boldsymbol{x}, \boldsymbol{d}) \leq \bar{v}_i, & i \in \mathcal{N}
\end{aligned}
$$

Projected Primal-Dual Gradient Dynamics (P-PDGD)

$$
\begin{aligned}
\dot{\boldsymbol{x}}_i &= k_x \left[\mathrm{Proj}_{\mathcal{X}_i}\left(\boldsymbol{x}_i - \alpha_x (\nabla c_i(\boldsymbol{x}_i) + \boldsymbol{\ell}_i) \right) - \boldsymbol{x}_i \right] \\
\dot{\lambda}_i^+ &= k_\lambda \left[\mathrm{Proj}_{\mathbb{R}_+}(\lambda_i^+ + \alpha_\lambda (v_i^{\mathrm{m}}(t) - \bar{v}_i)) - \lambda_i^+ \right] \\
\dot{\lambda}_i^- &= k_\lambda \left[\mathrm{Proj}_{\mathbb{R}_+}(\lambda_i^- + \alpha_\lambda (\underline{v}_i - v_i^{\mathrm{m}}(t))) - \lambda_i^- \right] \\
\boldsymbol{\ell}_i &= \sum_{j \in \mathcal{N}} (\lambda_j^+ - \lambda_j^-) \frac{\partial v_j(\boldsymbol{x}, \boldsymbol{d})}{\partial \boldsymbol{x}_i}
\end{aligned}
$$

● $i \in \mathcal{N}$

not implementable
due to **unknown system model**

Our Solution:

- Replace $v_i(\boldsymbol{x}, \boldsymbol{d})$ by real-time measurement $v_i^{\mathrm{m}}(t)$
- **Zeroth-order method** to estimate gradient $\frac{\partial v_j(\boldsymbol{x}, \boldsymbol{d})}{\partial \boldsymbol{x}_i}$

Optimal Voltage Control Design

Zeroth-order Method:

$$a\sin(\kappa t)+x \longrightarrow \boxed{v(x)} \longrightarrow v$$

Zeroth-order estimation **First-order gradient**

$$\frac{2}{a}\underbrace{v\big(x+a\sin(\kappa t)\big)}_{\text{real-time measurement } v^{\mathrm{m}}(t)}\sin(\kappa t) \approx \frac{dv(x)}{dx}$$

Projected Primal-Dual Gradient Dynamics (P-PDGD)

$$\dot{\boldsymbol{x}}_i = k_x\Big[\mathrm{Proj}_{\mathcal{X}_i}\big(\boldsymbol{x}_i - \alpha_x(\nabla c_i(\boldsymbol{x}_i)+\boldsymbol{\ell}_i)\big) - \boldsymbol{x}_i\Big]$$

$$\dot{\lambda}_i^+ = k_\lambda\Big[\mathrm{Proj}_{\mathbb{R}_+}\big(\lambda_i^+ + \alpha_\lambda(v_i^{\mathrm{m}}(t) - \bar{v}_i)\big) - \lambda_i^+\Big]$$

$$\dot{\lambda}_i^- = k_\lambda\Big[\mathrm{Proj}_{\mathbb{R}_+}\big(\lambda_i^- + \alpha_\lambda(\underline{v}_i - v_i^{\mathrm{m}}(t))\big) - \lambda_i^-\Big]$$

$$\boldsymbol{\ell}_i = \sum_{j\in\mathcal{N}}(\lambda_j^+ - \lambda_j^-)\frac{\partial v_j(\boldsymbol{x},\boldsymbol{d})}{\partial \boldsymbol{x}_i} \qquad \bullet\ i\in\mathcal{N}$$

not implementable
due to **unknown system model**

Our Solution:

- Replace $v_i(\boldsymbol{x},\boldsymbol{d})$ by real-time measurement $v_i^{\mathrm{m}}(t)$
- **Zeroth-order method** to estimate gradient $\frac{\partial v_j(\boldsymbol{x},\boldsymbol{d})}{\partial \boldsymbol{x}_i}$

Projected Primal-Dual Zeroth-Order Dynamics (P-PDZD)

$$\dot{\boldsymbol{x}}_i = k_x \left[\mathrm{Proj}_{\hat{\mathcal{X}}_i}\left(\boldsymbol{x}_i - \alpha_x(\nabla c_i(\boldsymbol{x}_i) + \boldsymbol{\ell}_i\right) - \boldsymbol{x}_i\right]$$

$$\dot{\lambda}_i^+ = k_\lambda \left[\mathrm{Proj}_{\mathbb{R}_+}(\lambda_i^+ + \alpha_\lambda(\mu_i - \bar{v}_i)) - \lambda_i^+\right]$$

$$\dot{\lambda}_i^- = k_\lambda \left[\mathrm{Proj}_{\mathbb{R}_+}(\lambda_i^- + \alpha_\lambda(\underline{v}_i - \mu_i)) - \lambda_i^-\right]$$

Projected Primal-Dual Dynamics

$$\dot{\boldsymbol{\ell}}_i = \frac{1}{\epsilon}\left[-\boldsymbol{\ell}_i + \frac{2}{a} \cdot N \cdot y_i(t) \sin(\boldsymbol{\kappa}_i t)\right]$$

$$\dot{\mu}_i = \frac{1}{\epsilon}\left[-\mu_i + v_i^{\mathrm{m}}(t)\right]$$

Zeroth-Order Gradient Learning

$$\dot{p}_i = \frac{1}{\epsilon_p} \cdot \sum_{ij \in \mathcal{E}} a_{ij}(y_i(t) - y_j(t))$$

$$y_i = (\lambda_i^+ - \lambda_i^-)\ v_i^{\mathrm{m}}(t) - p_i$$

Dynamic Average Consensus

- Each IBR unit $i \in \mathcal{N}$

Projected Primal-Dual Zeroth-Order Dynamics (P-PDZD)

local voltage measurement $v_i^{\mathrm{m}}(t)$

local communi. with neighbors $\big(y_j(t)\big)_{ij\in\mathcal{E}}$

$$\dot{\boldsymbol{x}}_i = k_x\Big[\mathrm{Proj}_{\hat{\mathcal{X}}_i}\big(\boldsymbol{x}_i - \alpha_x(\nabla c_i(\boldsymbol{x}_i) + \boldsymbol{\ell}_i\big) - \boldsymbol{x}_i\Big]$$

$$\dot{\lambda}_i^{+} = k_\lambda\Big[\mathrm{Proj}_{\mathbb{R}_+}\big(\lambda_i^{+} + \alpha_\lambda(\mu_i - \bar{v}_i)\big) - \lambda_i^{+}\Big]$$

$$\dot{\lambda}_i^{-} = k_\lambda\Big[\mathrm{Proj}_{\mathbb{R}_+}\big(\lambda_i^{-} + \alpha_\lambda(\underline{v}_i - \mu_i)\big) - \lambda_i^{-}\Big]$$

$$\dot{\boldsymbol{\ell}}_i = \frac{1}{\epsilon}\Big[-\boldsymbol{\ell}_i + \frac{2}{a}\cdot N \cdot y_i(t)\sin(\boldsymbol{\kappa}_i t)\Big]$$

$$\dot{\mu}_i = \frac{1}{\epsilon}\Big[-\mu_i + v_i^{\mathrm{m}}(t)\Big]$$

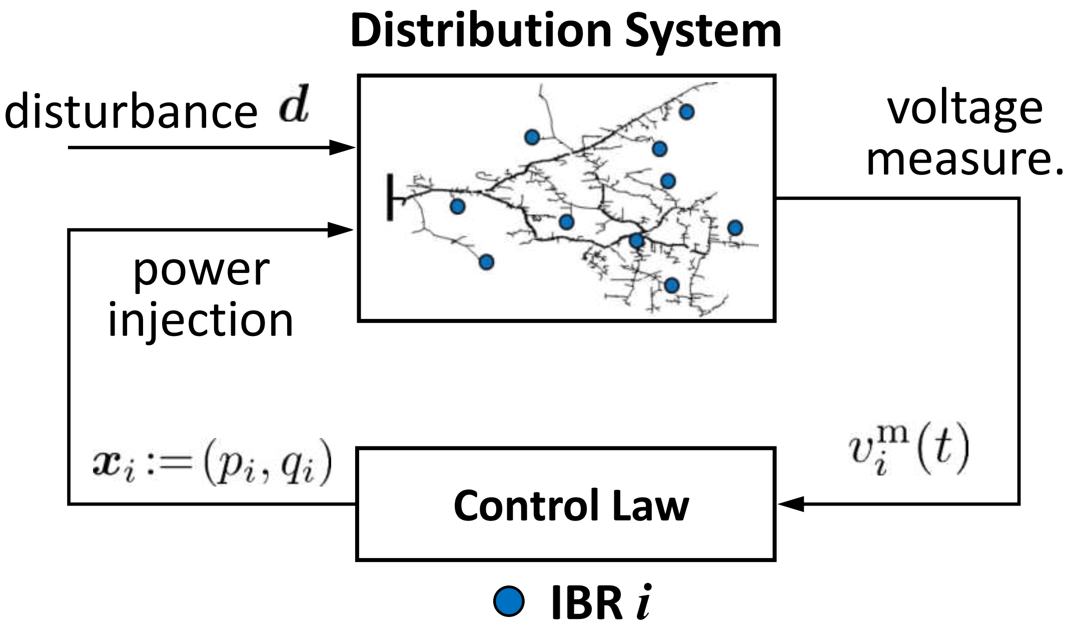
$$\dot{p}_i = \frac{1}{\epsilon_p}\cdot\sum_{ij\in\mathcal{E}} a_{ij}(y_i(t) - y_j(t))$$

$$y_i = (\lambda_i^{+} - \lambda_i^{-})\ v_i^{\mathrm{m}}(t) \ - p_i$$

Each IBR unit $i \in \mathcal{N}$

power reference setpoint

$\hat{\boldsymbol{x}}_i = (p_i^{ref}, q_i^{ref})$



Projected Primal-Dual Zeroth-Order Dynamics (P-PDZD)

$$\dot{\boldsymbol{x}}_i = k_x \left[\mathrm{Proj}_{\hat{\mathcal{X}}_i}\left(\boldsymbol{x}_i - \alpha_x(\nabla c_i(\boldsymbol{x}_i) + \boldsymbol{\ell}_i)\right) - \boldsymbol{x}_i\right]$$

$$\dot{\lambda}_i^+ = k_\lambda \left[\mathrm{Proj}_{\mathbb{R}_+}(\lambda_i^+ + \alpha_\lambda(\mu_i - \bar{v}_i)) - \lambda_i^+\right]$$

$$\dot{\lambda}_i^- = k_\lambda \left[\mathrm{Proj}_{\mathbb{R}_+}(\lambda_i^- + \alpha_\lambda(\underline{v}_i - \mu_i)) - \lambda_i^-\right]$$

$$\dot{\boldsymbol{\ell}}_i = \frac{1}{\epsilon}\left[-\boldsymbol{\ell}_i + \frac{2}{a} \cdot N \cdot y_i(t)\sin(\boldsymbol{\kappa}_i t)\right]$$

$$\dot{\mu}_i = \frac{1}{\epsilon}\left[-\mu_i + v_i^{\mathrm{m}}(t)\right]$$

$$\dot{p}_i = \frac{1}{\epsilon_p} \cdot \sum_{ij \in \mathcal{E}} a_{ij}(y_i(t) - y_j(t))$$

$$y_i = (\lambda_i^+ - \lambda_i^-)\ v_i^{\mathrm{m}}(t) \ - p_i$$

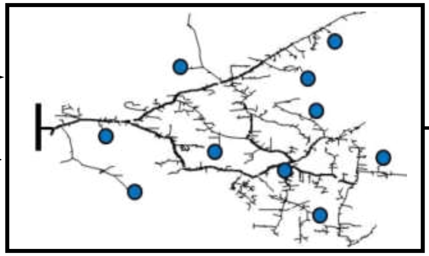
local voltage measurement $v_i^{\mathrm{m}}(t)$

local communi. with neighbors $(y_j(t))_{ij\in\mathcal{E}}$

power reference setpoint $\hat{\boldsymbol{x}}_i = (p_i^{ref}, q_i^{ref})$

Each IBR unit $i \in \mathcal{N}$

Distribution System



disturbance $\boldsymbol{d}$

power injection

voltage measure.

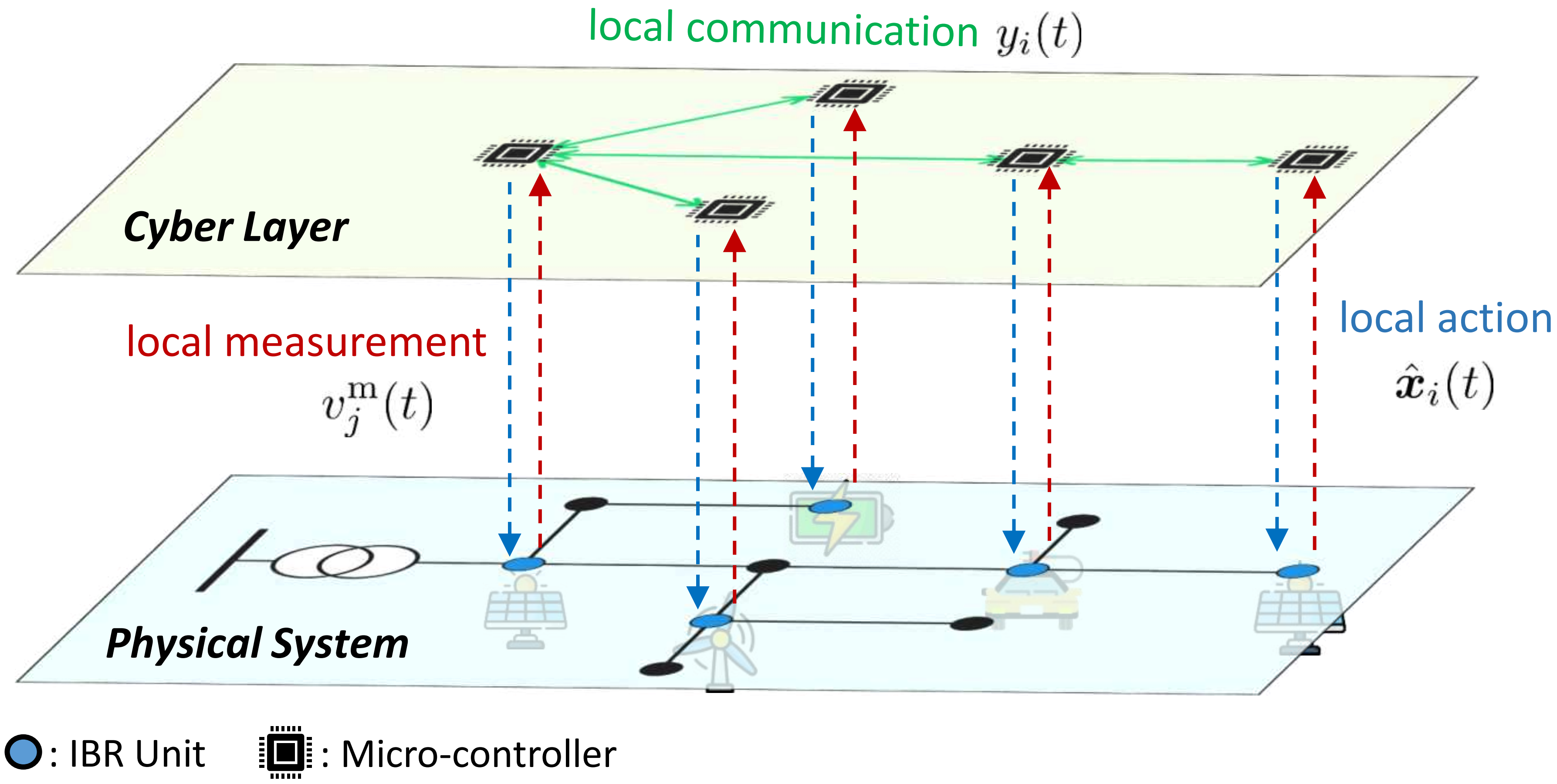
$\boldsymbol{x}_i := (p_i, q_i)$

$v_i^{\mathrm{m}}(t)$

P-PDZD

IBR i

Control Process Illustration



- ✓ **Optimality**
- ✓ **Safety**
- ✓ **Model-Free**
- ✓ **Distributed**
- ✓ **Self-adaptive**

Real-time physical system feedback + Zeroth-order gradient learning

Theoretical Guarantees

[**Chen**, et al, arXiv 2023] [**Chen**, et al, CDC 2021]

➢ (Semi-Global Practical Asymptotical Stability)

Theorem 1. (informal) Under assumptions of convexity, with feasible initial condition in a compact set, there exists a class-*KL* function β such that for any precision $\nu > 0$, with sufficiently small $(\epsilon, a, \varepsilon_\omega, \epsilon_p)$, the trajectory $\boldsymbol{z}(t)$ of the **P-PDZD** satisfies

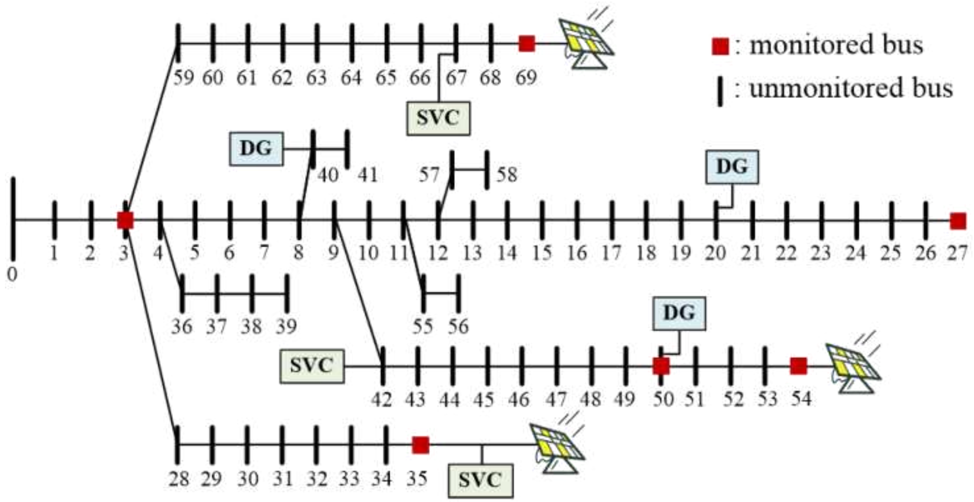
$$||\boldsymbol{z}(t)||_{\hat{\mathcal{A}}} \leq \beta(||\boldsymbol{z}(0)||_{\hat{\mathcal{A}}}, t) + \nu, \quad \forall t \geq 0.$$

➢ (Structural Robustness to Small Measurement Noise)

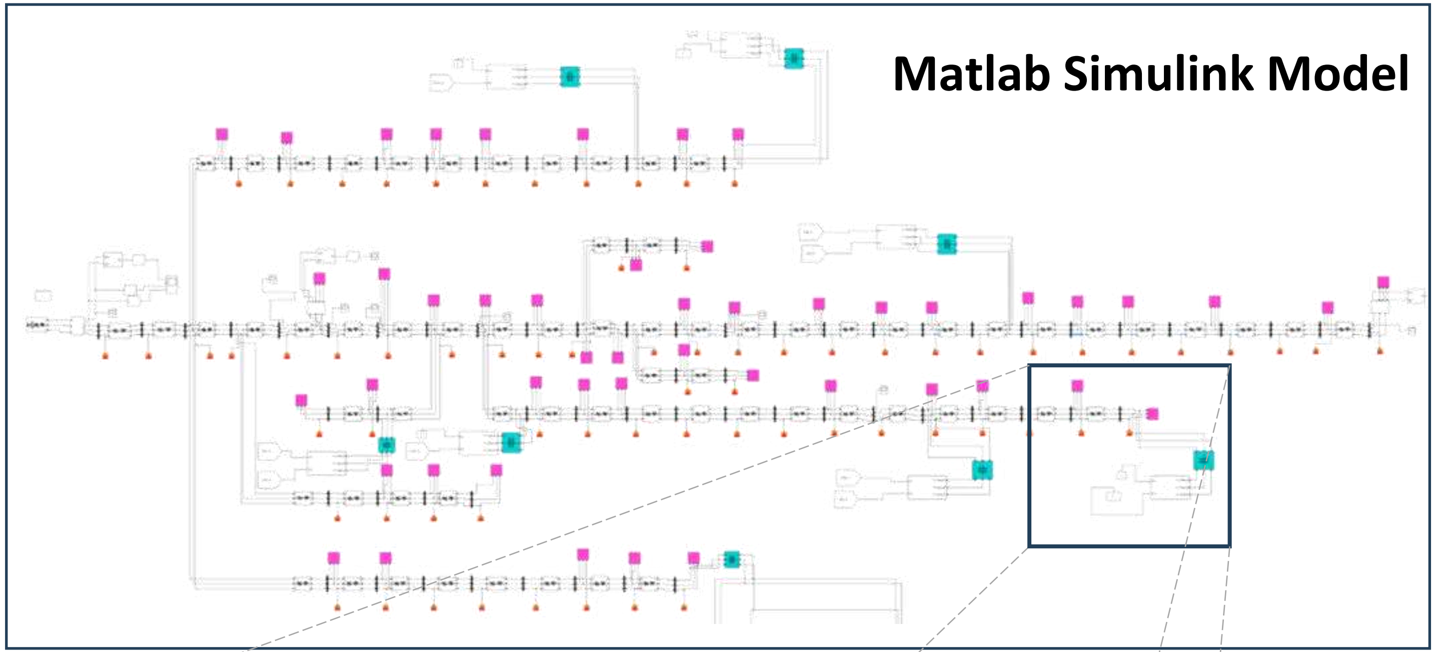
Theorem 2. (informal) Under the same conditions in Theorem 1, there exists $\rho^* > 0$ such that for any measurement noise $\boldsymbol{d}$ with $\sup_{t\geq 0} ||\boldsymbol{d}(t)|| \leq \rho^*$, the trajectory $\boldsymbol{z}(t)$ of the **P-PDZD** with additive measurement noise satisfies

$$||\boldsymbol{z}(t)||_{\hat{\mathcal{A}}} \leq \beta(||\boldsymbol{z}(0)||_{\hat{\mathcal{A}}}, t) + 2\nu, \quad \forall t \geq 0.$$

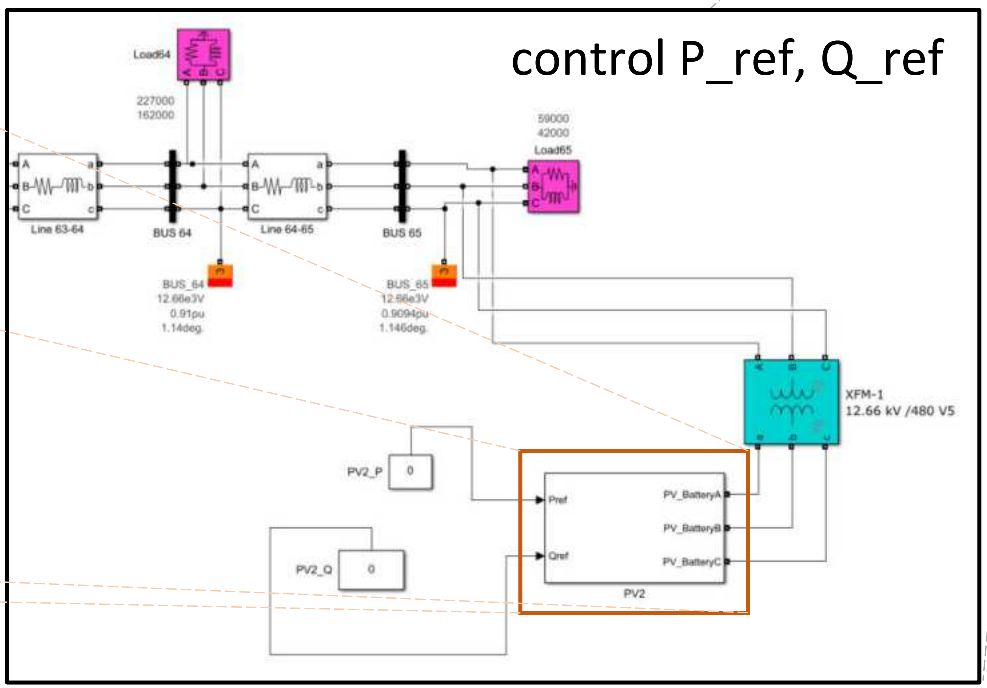
Simulation



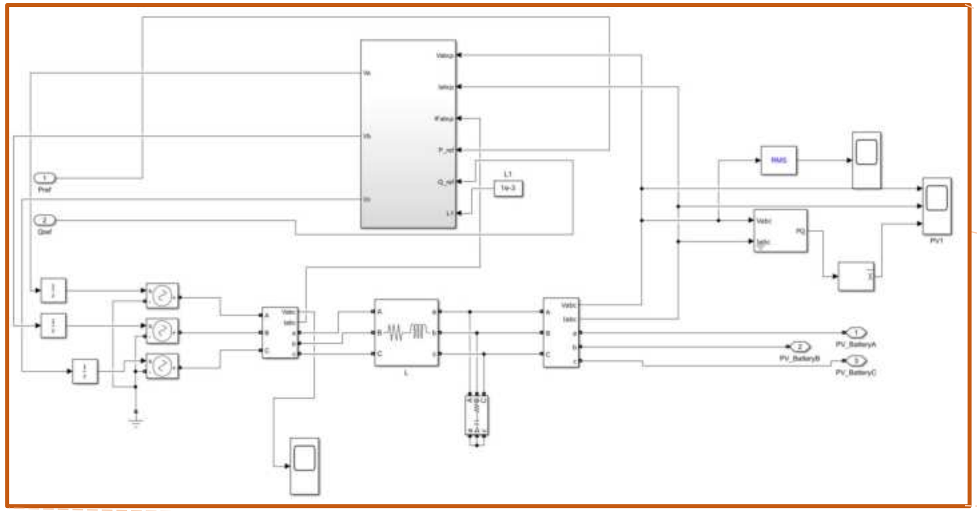
3-Phase IEEE 69-Bus Distribution System



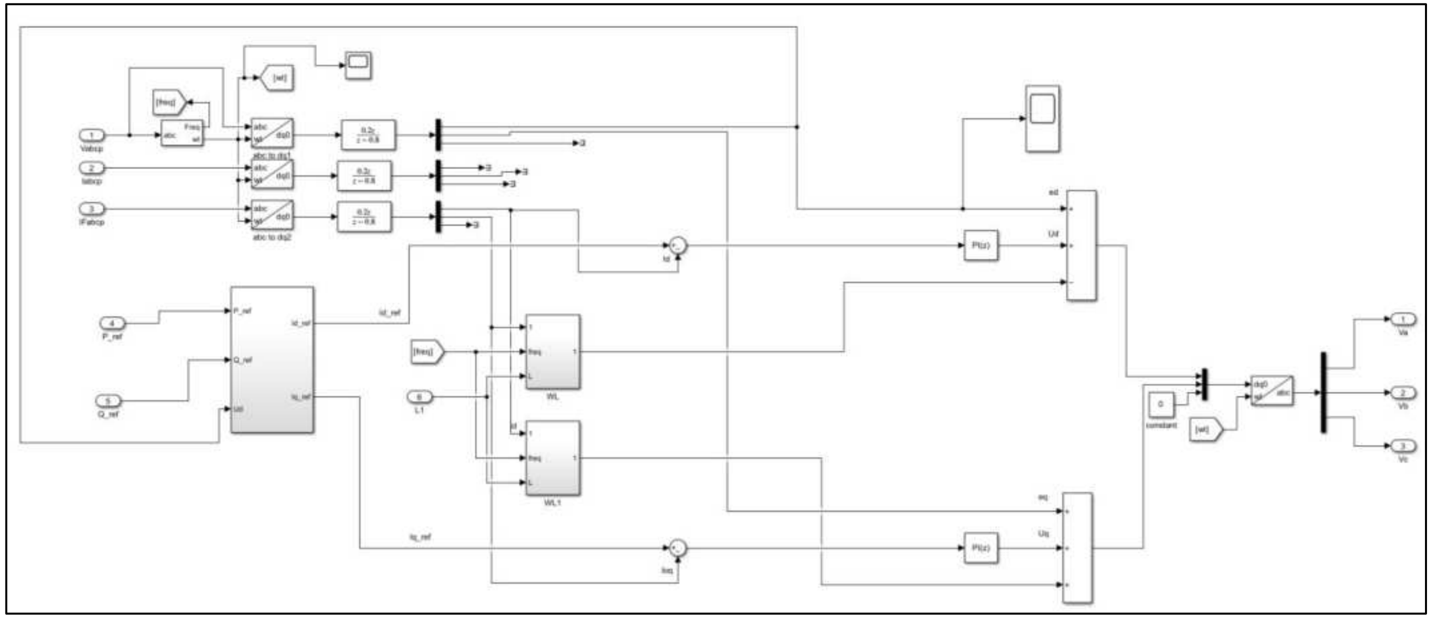
Matlab Simulink Model



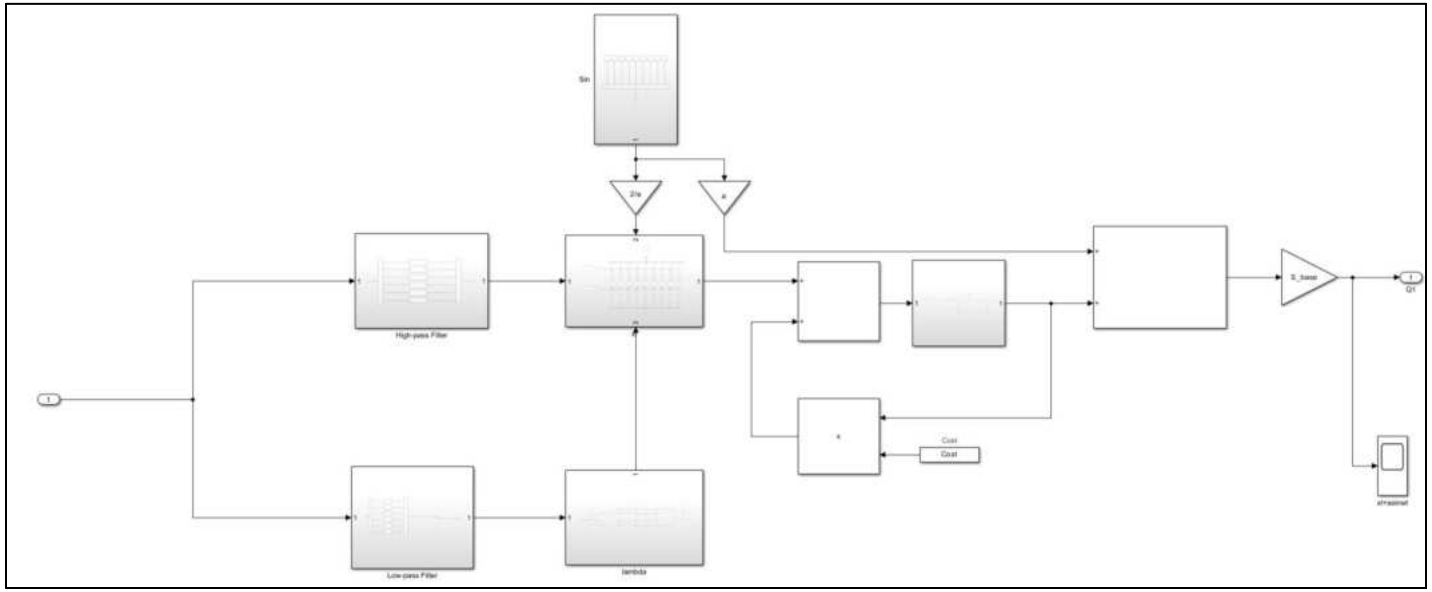
control P_ref, Q_ref



high-fidelity EMT model



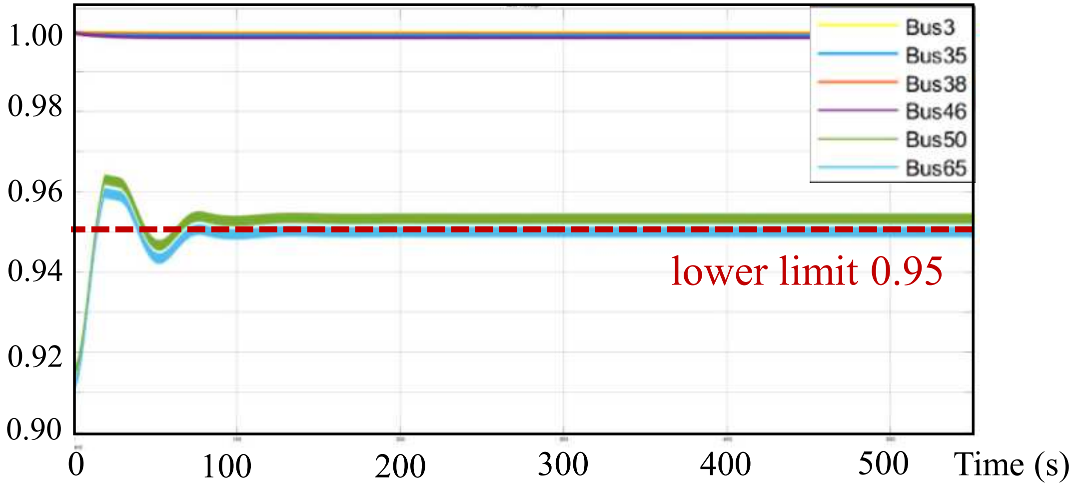
PV Inverter Inner Control Loops
(Grid-Following)



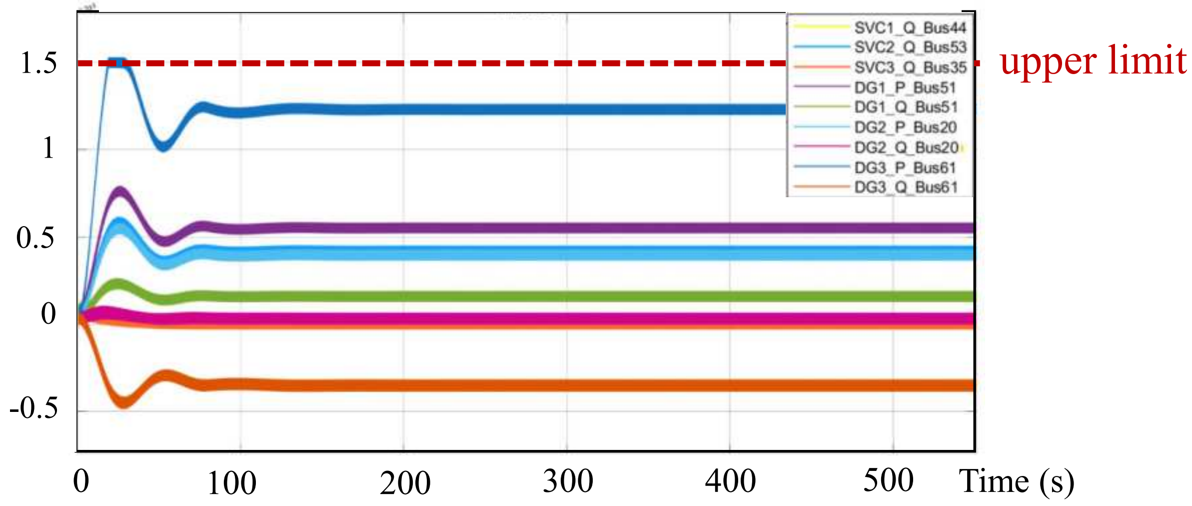
Projected Primal-Dual Zeroth-Order
Control Algorithm Implementation

Case 1. Step Power Disturbance

Bus Voltage Magnitude (p.u.)

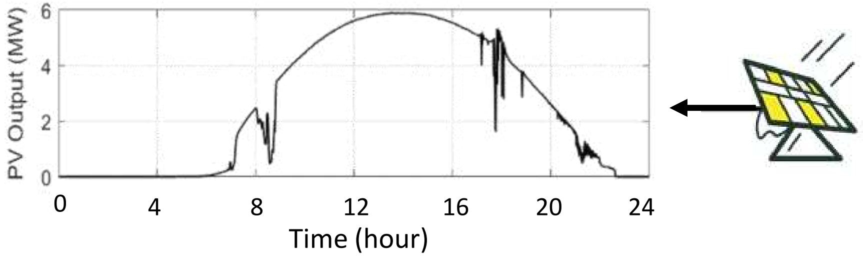


Flexible IBR Power Outputs (MW/Mvar)

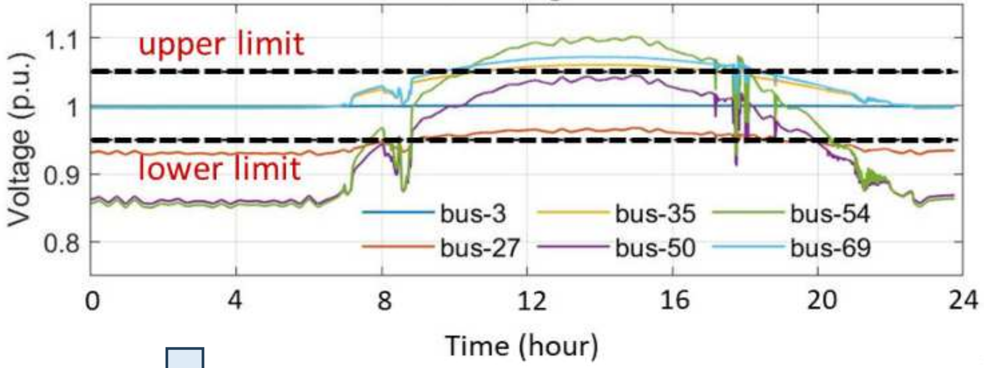


Case 3. Continuous Power Disturbance

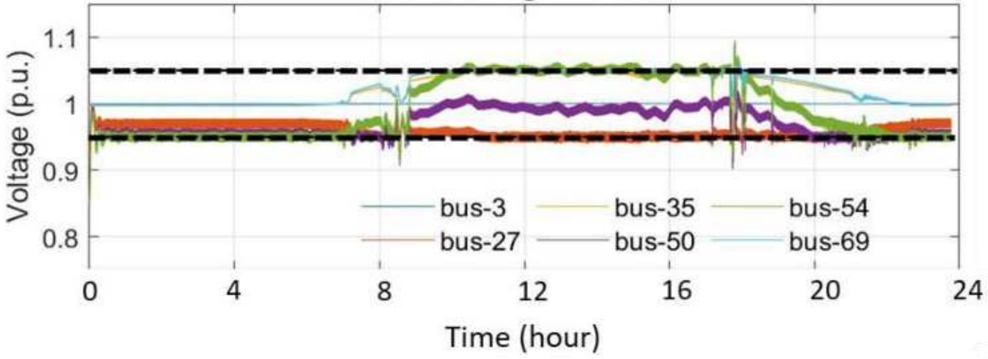
Real time-varying solar generation



without voltage control



with voltage control



Projected Primal-Dual Zeroth-Order Method

Feedback Optimization

Zeroth-order Gradient Learning

(Physical System Dynamics + Control Law = Optimization Algorithm)

➢ **Real-time model-free optimal control** of complex (multi-agent) physical systems

✓ Unknown system model

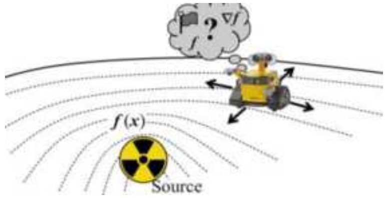
✓ Scalability ✓ Safety

✓ Performance guarantees

Building Energy Control

Wind Farm Control

Source Seeking



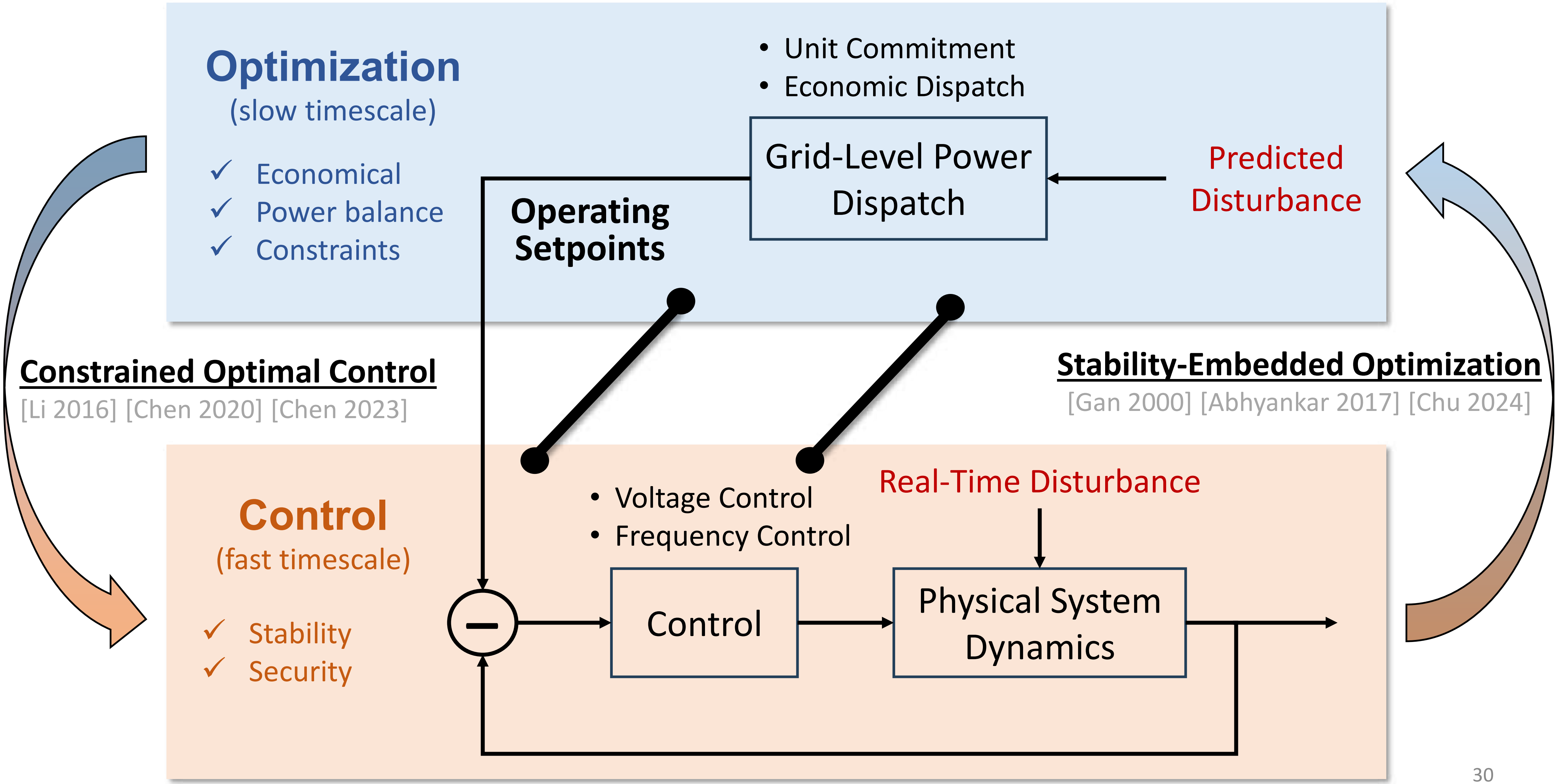
Resource Allocation

[1] **X. Chen**, J. I. Poveda, N. Li, "Continuous-Time Zeroth-Order Dynamics with Projection Maps: Model-Free Feedback Optimization with Safety Guarantees", 2024. (accepted by IEEE Trans. Automatic Control)

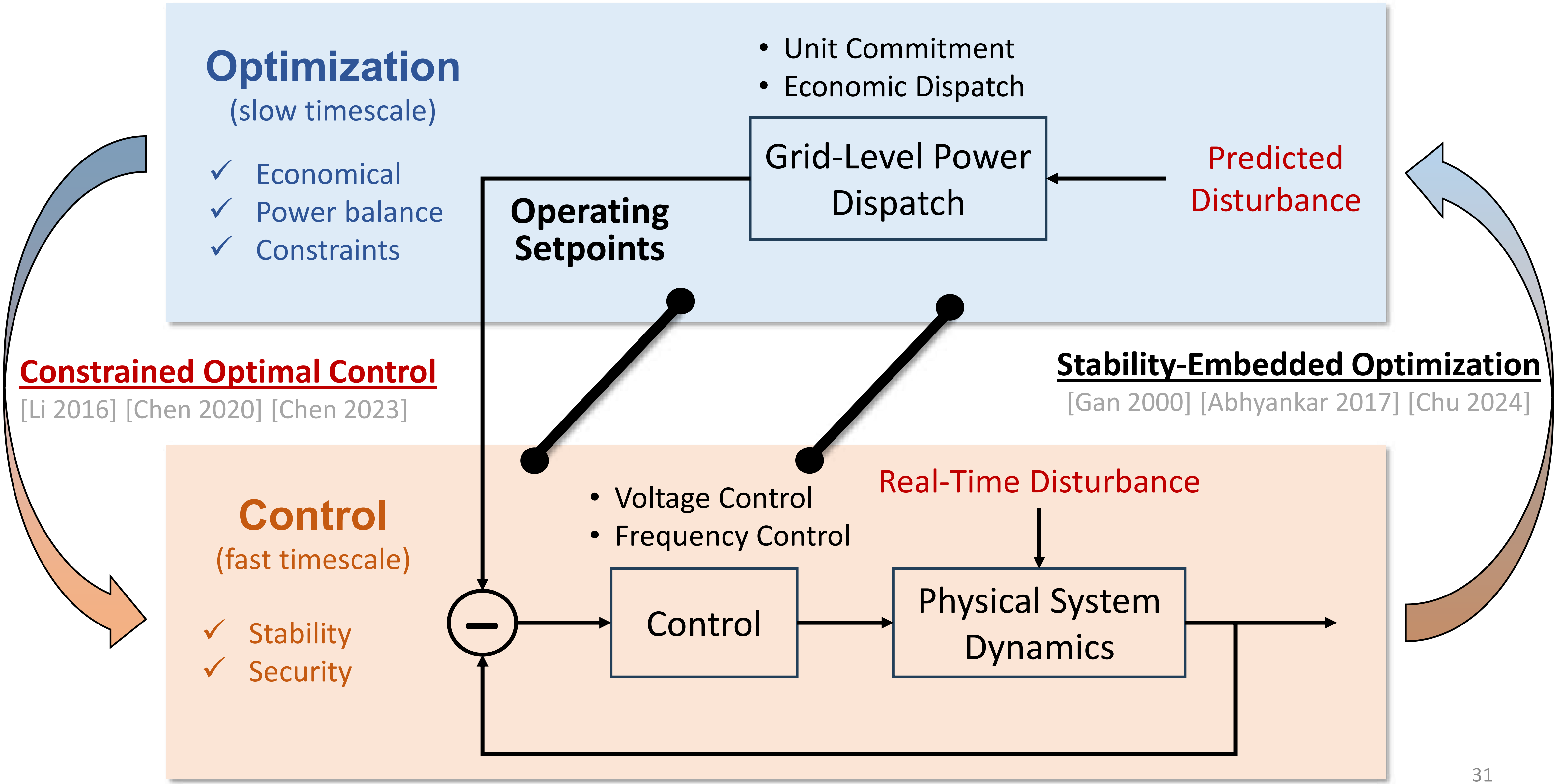
Looking Forward . . .

- Incorporate available system model info (model-based + data-driven)
- Dynamics modeling and identification of IBR-dominant systems
- (Large-signal) stability for nonlinear power system dynamics
- Field testing
- **Redesign the grid-level coordination structure?**

Grid-Level IBR Coordination Structure



Grid-Level IBR Coordination Structure



This Talk

Coordinate large-scale IBRs for grid-level optimal control

Part I.
Optimal Voltage Control

[**Chen**, Li, CDC, 2021] [**Chen**, Li, IEEE TAC, 2024]

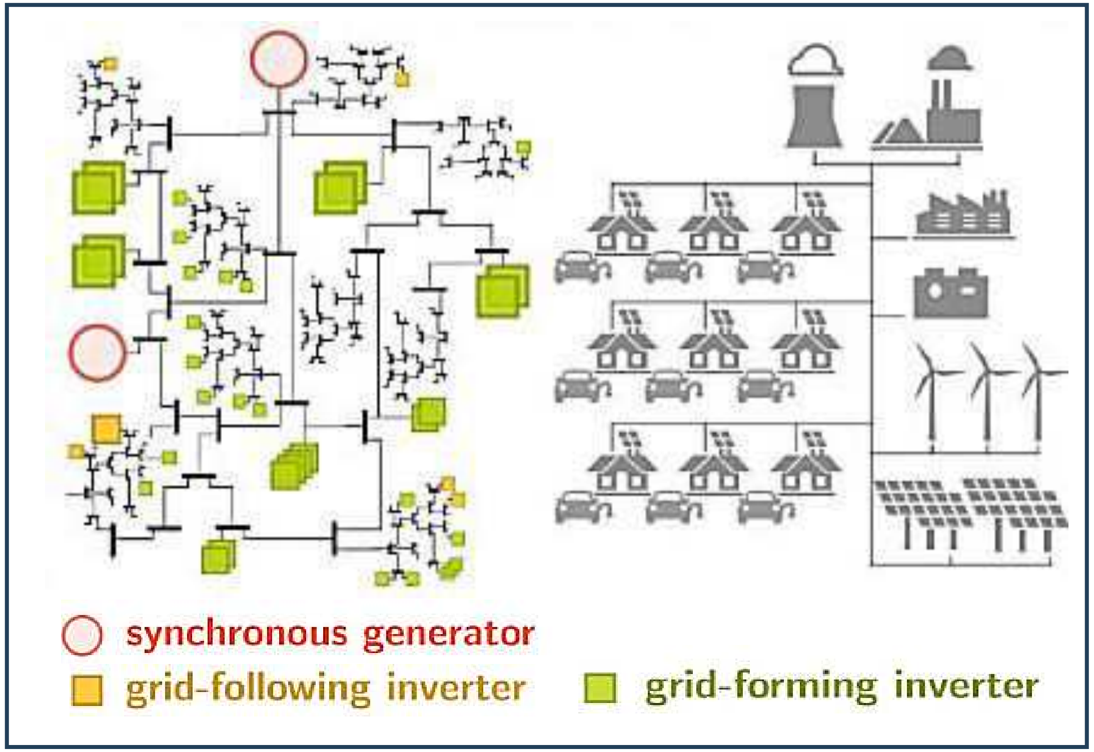
Part II.
Optimal Frequency Control

[**Chen**, Li, CCTA, 2018] [**Chen**, Li, IEEE TCNS, 2021]

Data-Driven **+** Distributed ⟶ Unknown Model **+** Scalability

Leverage specific problem structures & Feedback Optimization

Future Power Converter-Dominant Grid



- Lower inertia, faster dynamics
+ More controllable IBRs with fast response
- Scalability for controlling massive IBRs
- Unknown system dynamics models

Our Control Goals

Coordinate IBRs by controlling their **power setpoints** for **optimal frequency control**

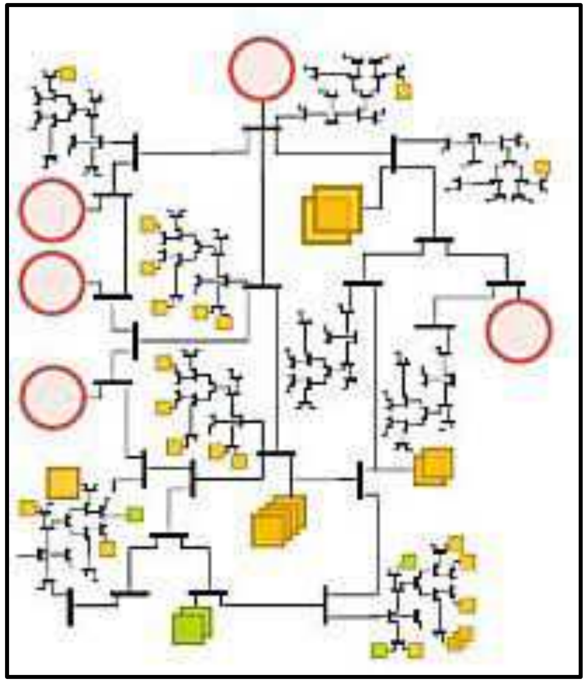
✓ **Primary + Secondary + Tertiary** freq. control
(stabilize freq. **+** restore nominal freq. **+** economic dispatch)

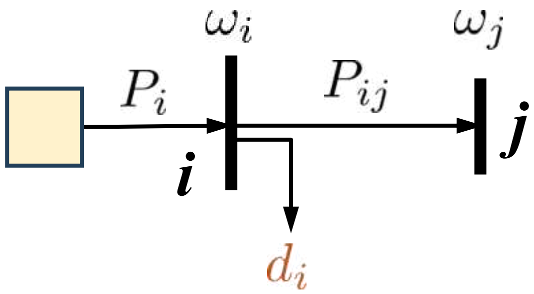
✓ Satisfy **IBR power limits** and **network constraints**

✓ **Distributed control** to enable scalability

✓ **Model-free control**
(no need grid dynamics model and disturbance info.)

Physical System Frequency Dynamics





- **DC power flow dynamics:**

$$\dot{P}_{ij} = B_{ij}\left(\omega_i - \omega_j\right)$$

- **Nodal power balance:**

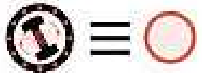
$$P_i = d_i + \sum_{j:ij\in\mathcal{E}} P_{ij}$$

uncontrollable net load (**disturbance**)

branch flow

Control IBR power setpoints (P_i^M, P_i^L) for frequency regulation

- **Device-level frequency response dynamics**

 synchronous generator bus $i \in \mathcal{N}_G$

$$M_i\dot{\omega}_i + D_i\omega_i = P_i^G - P_i$$

generator inertia, damping coefficient, frequency deviation, mechanical power, output power

 grid-forming inverter bus $i \in \mathcal{N}_M$

$$k_i^M\omega_i = \frac{\alpha_i}{\alpha_i + s}\left(P_i^M - P_i\right)$$

droop gain, low-pass filter, power setpoint, output power

$$\Longleftrightarrow \quad \frac{1}{\alpha_i}\dot{\omega}_i + k_i^M\omega_i = P_i^M - P_i$$

 grid-following inverter bus $i \in \mathcal{N}_L$

$$P_i = P_i^L - k_i^L\omega_i$$

output power, power setpoint, droop gain

Optimal Frequency Control (OFC)

Economic Dispatch

OFC

$$\text{Obj. } \min_{\boldsymbol{P},\boldsymbol{\theta}} \sum_{i\in\mathcal{N}_M} c_i^M(P_i^M) + \sum_{i\in\mathcal{N}_L} c_i^L(P_i^L)$$

$$\text{s.t. } P_i^M \in [\underline{P}_i^M, \overline{P}_i^M], \quad P_i^L \in [\underline{P}_i^L, \overline{P}_i^L]$$

$$P_i^M = d_i + \sum_{j:i\to j} B_{ij}(\theta_i - \theta_j) - \sum_{k:k\to i} B_{ki}(\theta_k - \theta_i), \quad \forall i \in \mathcal{N}_M$$

$$P_i^L = d_i + \sum_{j:i\to j} B_{ij}(\theta_i - \theta_j) - \sum_{k:k\to i} B_{ki}(\theta_k - \theta_i), \quad \forall i \in \mathcal{N}_L$$

$$\underline{P}_{ij} \le B_{ij}(\theta_i - \theta_j) \le \overline{P}_{ij}, \quad \forall ij \in \mathcal{E}$$

$$P_i^{M*}, P_i^{L*}$$

add ω_i

and auxiliary constraints

$$\text{Obj. } \min_{\boldsymbol{P},\boldsymbol{\theta},\boldsymbol{\omega}} \sum_{i\in\mathcal{N}_M} \left(c_i^M(P_i^M) + \frac{1}{2} k_i^M \omega_i^2 \right) + \sum_{i\in\mathcal{N}_L} \left(c_i^L(P_i^L) + \frac{1}{2} k_i^L \omega_i^2 \right)$$

$$\text{s.t. } P_i^M \in [\underline{P}_i^M, \overline{P}_i^M], \quad P_i^L \in [\underline{P}_i^L, \overline{P}_i^L]$$

$$P_i^M = k_i^M \omega_i + d_i + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}, \quad \forall i \in \mathcal{N}_M$$

$$P_i^L = k_i^L \omega_i + d_i + \sum_{j:i\to j} P_{ij} - \sum_{k:k\to i} P_{ki}, \quad \forall i \in \mathcal{N}_L$$

$$P_i^M = d_i + \sum_{j:i\to j} B_{ij}(\psi_i - \psi_j) - \sum_{k:k\to i} B_{ki}(\psi_k - \psi_i), \quad \forall i \in \mathcal{N}_M$$

$$P_i^L = d_i + \sum_{j:i\to j} B_{ij}(\psi_i - \psi_j) - \sum_{k:k\to i} B_{ki}(\psi_k - \psi_i), \quad \forall i \in \mathcal{N}_L$$

$$\underline{P}_{ij} \le B_{ij}(\psi_i - \psi_j) \le \overline{P}_{ij},$$

Modified OFC

$$P_i^{M*}, P_i^{L*}, \omega_i^* = 0$$

Solved by

dual variables λ

Projected Primal-Dual Gradient Algorithm

Optimal Frequency Control (OFC)

Projected Primal-Dual Gradient Algorithm

$$\frac{1}{\alpha_i}\dot{\omega}_i = P_i^M - k_i^M \omega_i - d_i - \sum_{j:i\to j} P_{ij} + \sum_{k:k\to i} P_{ki} \quad i \in \mathcal{N}_M$$

$$0 = P_i^L - k_i^L \omega_i - d_i - \sum_{j:i\to j} P_{ij} + \sum_{k:k\to i} P_{ki} \quad i \in \mathcal{N}_L$$

$$\dot{P}_{ij} = B_{ij}\left(\omega_i - \omega_j\right) \quad ij \in \mathcal{E}$$

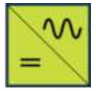
$$\dot{P}_i^M = f_i^M\left(P_i^M, \omega_i, (P_{ij})_{i\leftrightarrow j}, (\lambda_j)_{i\leftrightarrow j}\right) \quad i \in \mathcal{N}_M$$

$$\dot{P}_i^L = f_i^L\left(P_i^L, \omega_i, (P_{ij})_{i\leftrightarrow j}, (\lambda_j)_{i\leftrightarrow j}\right) \quad i \in \mathcal{N}_L$$

Optimal Frequency Control (OFC)

Physical Power System Dynamics

$$
\begin{aligned}
\frac{1}{\alpha_i}\dot{\omega}_i &= P_i^M - k_i^M \omega_i - d_i - \sum_{j:i\to j} P_{ij} + \sum_{k:k\to i} P_{ki} && i \in \mathcal{N}_M \\
0 &= P_i^L - k_i^L \omega_i - d_i - \sum_{j:i\to j} P_{ij} + \sum_{k:k\to i} P_{ki} && i \in \mathcal{N}_L \\
\dot{P}_{ij} &= B_{ij}\left(\omega_i - \omega_j\right) && ij \in \mathcal{E}
\end{aligned}
$$

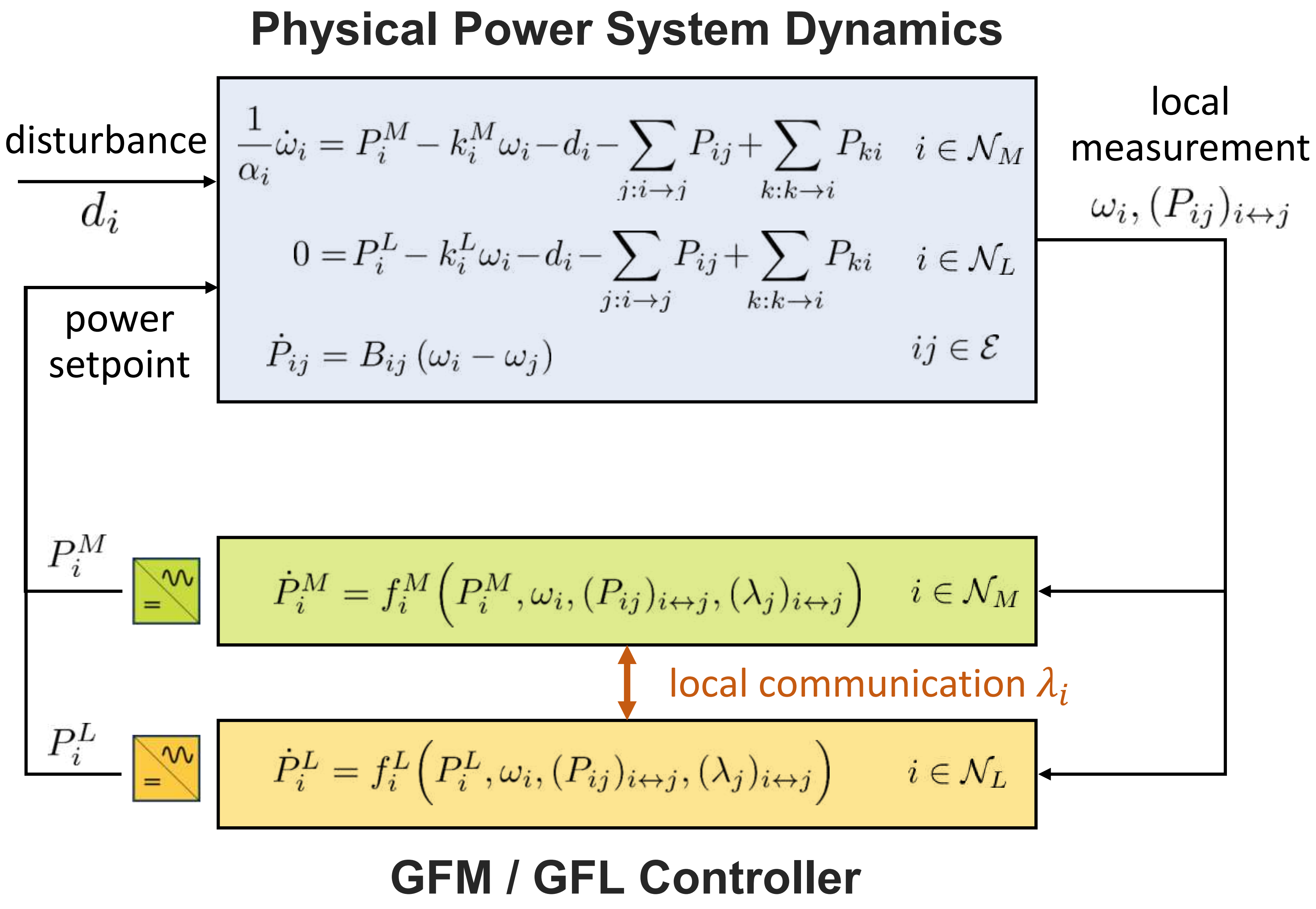


$$
\dot{P}_i^M = f_i^M\Big(P_i^M, \omega_i, (P_{ij})_{i\leftrightarrow j}, (\lambda_j)_{i\leftrightarrow j}\Big) \qquad i \in \mathcal{N}_M
$$

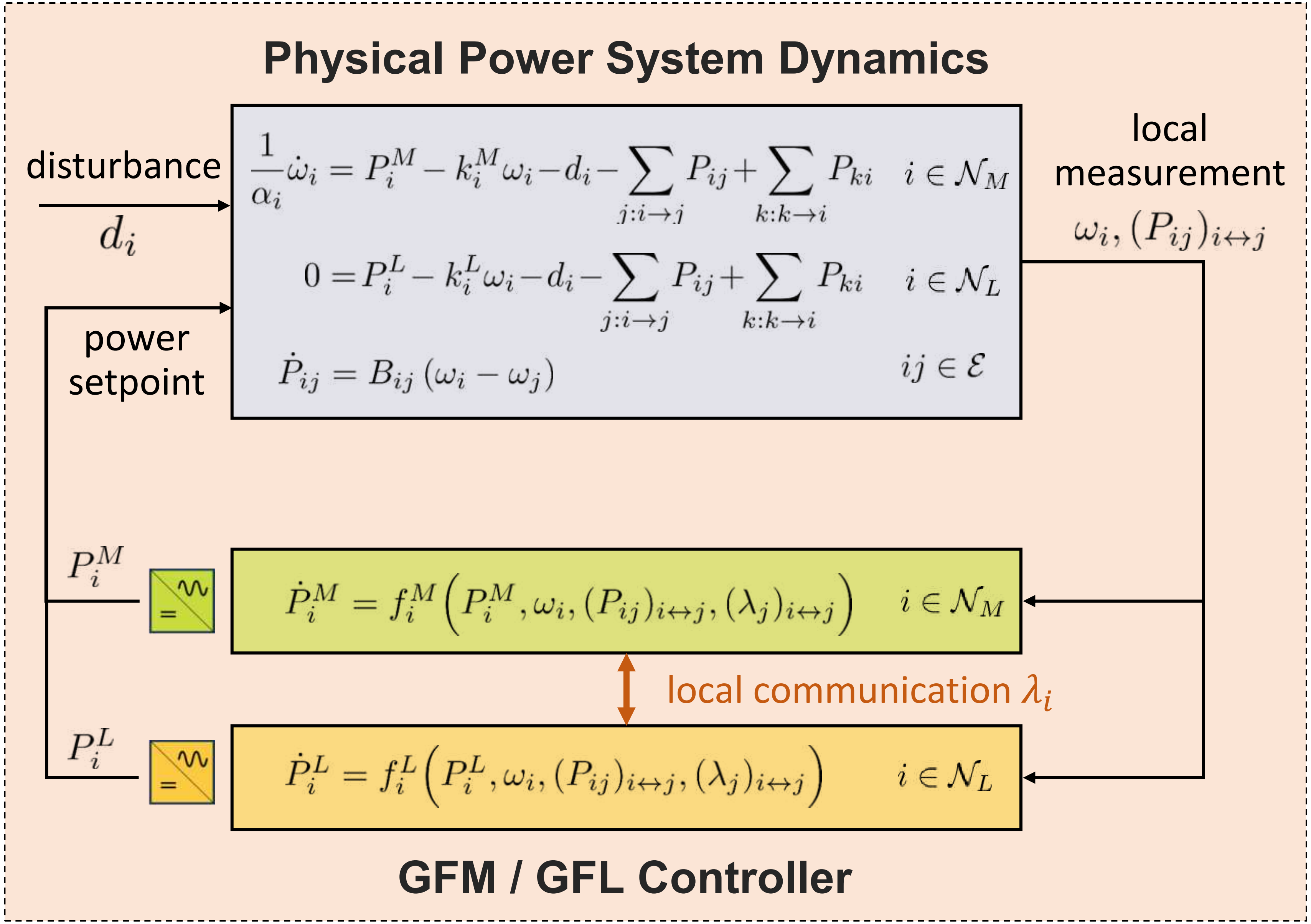
$$
\dot{P}_i^L = f_i^L\Big(P_i^L, \omega_i, (P_{ij})_{i\leftrightarrow j}, (\lambda_j)_{i\leftrightarrow j}\Big) \qquad i \in \mathcal{N}_L
$$

GFM / GFL Controller

Optimal Frequency Control (OFC)



Optimal Frequency Control (OFC)



$=$ **Primal-Dual Gradient Solution Dynamics**

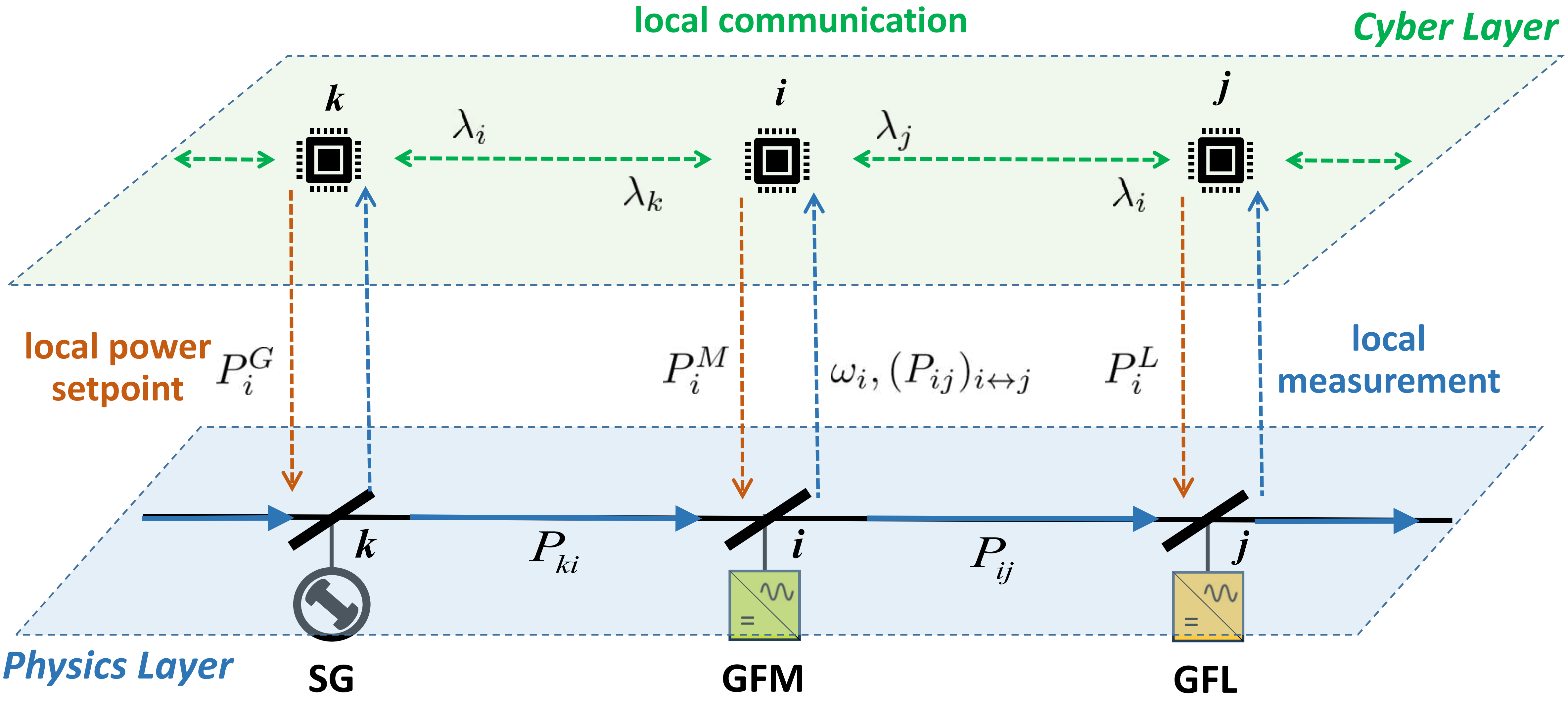
↓

$$P_i^{M*}, P_i^{L*}, \omega_i^* = 0$$

- ✓ *Optimal decision*
- ✓ *Restore nominal frequency*
- ✓ *Distributed control*
- ✓ *No need system model*

Control Process Illustration

Closed-loop Feedback system



Real-time system feedback + physical dynamics structure

Theoretical Guarantees

❖ Asymptotical Convergence

Theorem 1. (informal) Suppose that the cost function is strictly convex and the OFC problem is feasible. Then the proposed control algorithm together with the power system dynamics asympotically converges to an optimal solution of OFC problem.

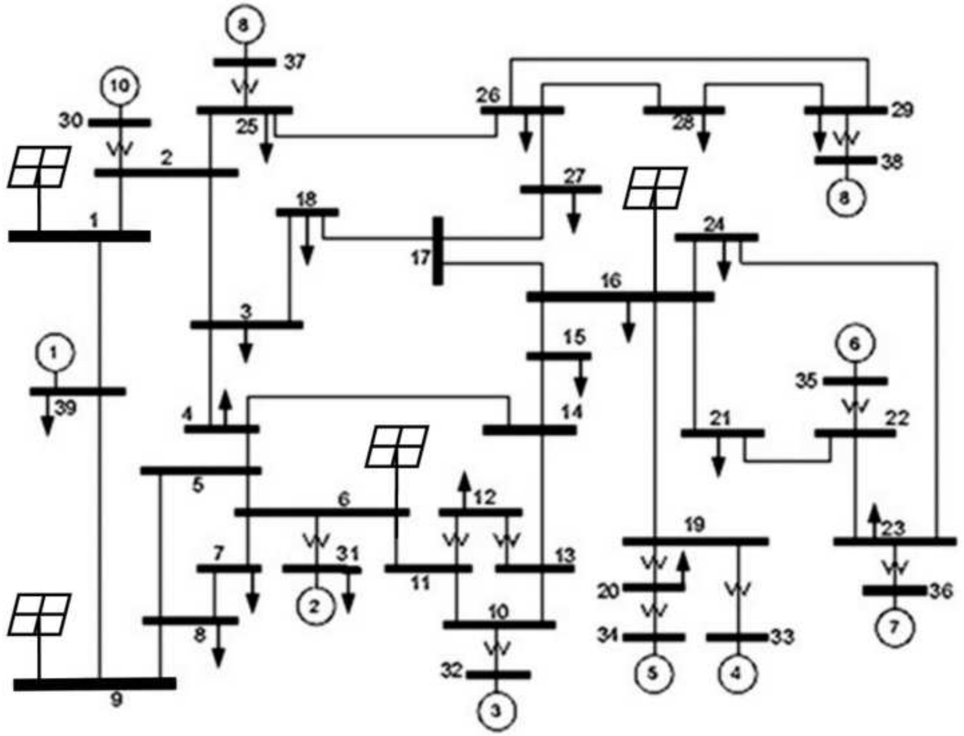
❖ Dynamic Tracking Performance

Theorem 2. (informal) Suppose that the cost function is strongly convex and the OFC problem is feasible. The tracking error of the actual system dynamics under continuous disturbance is bounded in the sense that $||\boldsymbol{x}(t)-\boldsymbol{x}^*(t)|| \leq e^{-\rho t}||\boldsymbol{x}(0)-\boldsymbol{x}^*(0)|| + (1-e^{-\rho t})\frac{b_x+b_g}{\rho}$.

Xin Chen, Changhong Zhao, Na Li, "Distributed Automatic Load-frequency Control with Optimality in Power Systems," IEEE Transactions on Control of Network Systems, vol. 8, no. 1, pp. 307-318, March 2021.

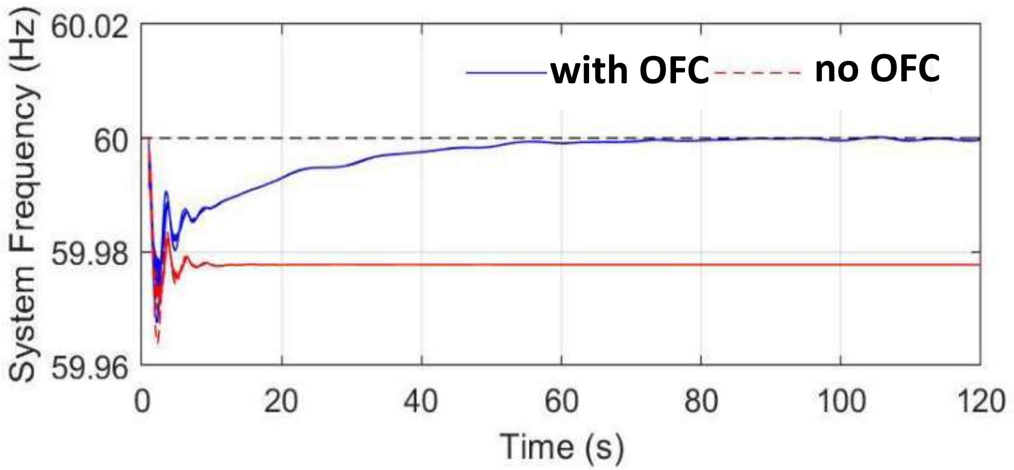
Simulations

- Simulations are run on **Power System Toolbox** [1]
  - ✓ AC power flow model.
  - ✓ Classic two-axis sub-transient generator model.
  - ✓ IEEE Type DC1 excitation system model.

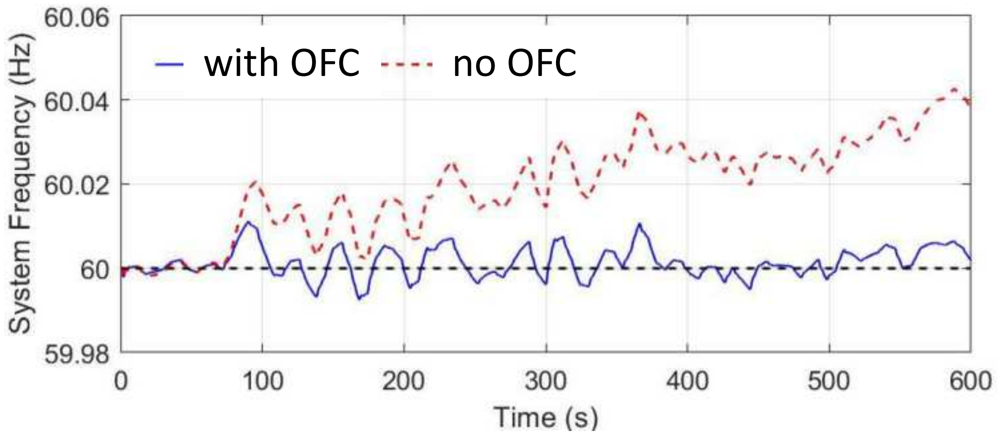


The 39-bus New England Power Network.

Case 1: Step Power Change



Case 2: Continuous Power Change



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